What are reference rates for?

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January 13, 2020

Abstract

Reference rates like LIBOR were designed to reflect banks' full cost of funds, including credit premia. With publication of LIBOR expected to cease soon, regulators have recommended a shift towards risk free reference rates. This paper examines the role of reference rates in loan markets using a simple, tractable model of maturity transformation. The model highlights that reference rates that incorporate credit risk may serve an important purpose in credit markets: facilitating contracts that transfer funding risk. While risk free reference rates may be appropriate for derivatives, their use may lead to unintended consequences in credit markets.

JEL Classification Numbers: G21, G32, G18

*E-mail: dkirti@imf.org. I would like to thank, without implication, Philippe Aghion, Jerry Green, Oliver Hart, Benjamin Hébert, David Jones, Eric Maskin, Andrei Shleifer, Jeremy Stein, Paul Tucker, and seminar participants at the CFTC for helpful conversations. The views expressed herein are those of the author and should not be attributed to the IMF, its Executive Board, or its management.

1 Introduction

Much attention has been drawn to reference rates such as the London Interbank Offered Rate (LIBOR), and other similar 'IBORs', over the past decade. During the global financial crisis, it became clear that these reference rates—tied to increasingly dormant interbank funding markets and frequently manipulated—were problematic. Regulators have since recommended a shift towards 'risk-free reference rates' (RFRs), partly motivated by the widespread use of reference rates in derivatives markets (Duffie & Stein 2015). In line with this recommendation, alternative benchmark rates have been chosen in key jurisdictions that are either explicitly based on transactions backed by high quality collateral (such as the Secured Overnight Financing Rate, SOFR, in the US) or aim to include minimal credit risk (such as the Sterling Overnight Index Average, SONIA, in the UK). Publication of LIBOR is expected to cease in 2021.

Reference rates continue to underpin large credit markets. The shift to RFRs will have important implications for the allocation of risk in these markets. Consider the experience of the global financial crisis (Figure 1). As bank credit risk rose, LIBOR increased too, raising the cost of funds for existing borrowers. Consequently, some uncertainty about the realized cost of funds—funding risk—was borne by these existing borrowers rather than banks. Banks were also able to accommodate a spike in demand for liquidity (Ivashina & Scharfstein 2010). Over the same period, interest rates fell for the types of secured transactions used to calculate SOFR.[1]

IBORs were designed to reflect banks' full cost of borrowing, including credit risk premia. There have been extensive discussions regarding what 'benchmark replacement adjustment' should be applied to RFRs to account for credit risk. At the time of writing, a consensus for time-invariant credit spread adjustments seems to be emerging, although there are also attempts to construct new reference rates that incorporate lender credit risk (see Section 2). Had LIBOR been replaced by RFRs with fixed credit spread adjustments during the global financial crisis, banks would have had to bear a greater portion of funding risk themselves.

[1]Figure 1 shows interest rates for general collateral repo transactions. SOFR includes general collateral, triparty, as well as cleared bilateral treasury repo transactions, with indicative estimates available 2014 onwards.

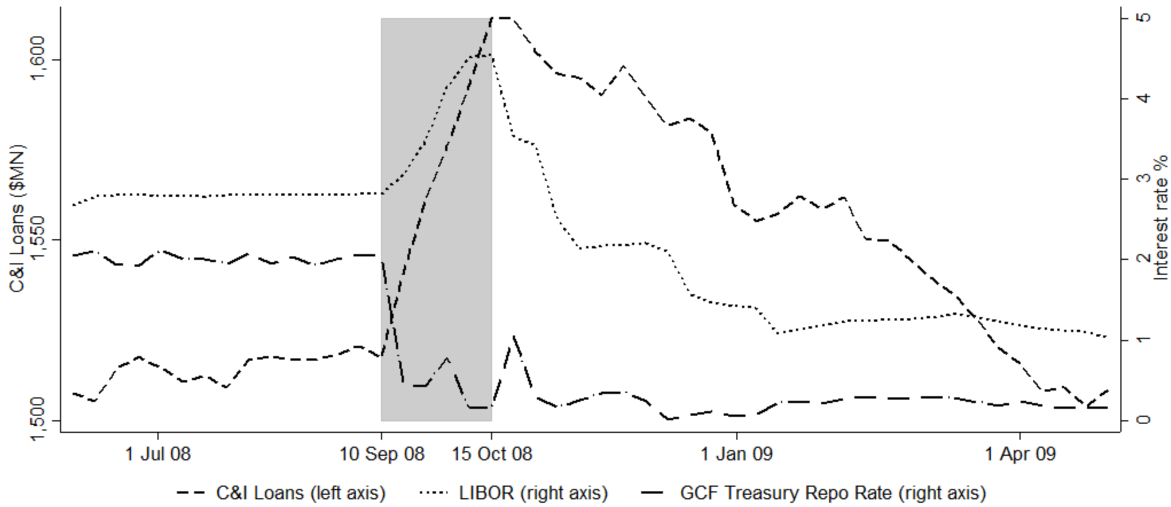

Figure 1: *Funding risk during the global financial crisis (Sources: FRED, Bloomberg, DTCC)*

This paper examines the role of reference rates in loan markets using a simple, tractable model of maturity transformation. The model incorporates three key frictions. First, lenders must use short-term liabilities to fund long-term loans—a stronger version of the assumption that short-term liabilities that meet demand for liquidity and safety are cheaper (Diamond & Dybvig 1983, Gorton & Pennacchi 1990, Gorton 2010, Stein 2012)—leading to maturity mismatch. Next, banks' funding costs are not directly contractible: borrowers are unable to observe these costs directly, and banks may not wish to disclose them. Reference rates serve the purpose of mitigating this contractual incompleteness, facilitating contingent contracts. Finally, financial frictions lead to effective risk aversion (Froot, Scharfstein & Stein 1993, Froot & Stein 1998). In combination, these three frictions lead to a setting in which funding risk is meaningful, and contracting parties value the contingent contracts, facilitated by reference rates, that can help manage this risk.

This framework permits analysis of how the availability of and properties of reference rates affect credit markets. Using an asset demand approach on both sides of the market, I solve for the equilibrium quantity of and interest rate on loans under different institutional arrangements. In the absence of reference rates, lenders can only offer fixed interest rates, and must bear funding risk themselves. Given sufficiently accurate reference rates, floating-rate loan contracts allow funding risk to be transferred to borrowers, lowering loan rates. Whether welfare increases with this arrangement depends on the cost to firms of bearing interest-rate risk.

Viewed through the lens of the model, the prevalence of floating-rate corporate lending suggests that the transfer of interest-rate risk from banks to firms facilitated by reference rates is welfare-enhancing. Recent empirical work suggests that banks manage their interest-rate exposure in part by matching the structure of their assets with their liabilities (Drechsler, Savov & Schnabl 2018, Kirti 2019). Consistent with the model, banks seem to manage interest-rate risk partly by transferring some of their exposure to their borrowers.[2]

In credit markets, reference rates can help facilitate contracts contingent on lenders' full realized cost of funds. Floating rate loans only improve on fixed rate loans in the model if the reference rate is sufficiently highly correlated with bank funding costs. Indeed, it is common for loan contracts to explicitly refer to the need for reference rates that accurately reflect lenders' actual cost of funding (see Section 2).[3] To do so, reference rates must link tightly with banks' full funding costs, including credit risk premia.

RFRs with fixed credit spread adjustments may be appropriate for interest-rate derivatives markets. However, their use may lead to unintended consequences in credit markets. Lenders will need to bear more funding risk themselves in the absence of time-varying credit spread adjustments or other markets to transfer funding risk. When bank credit risk spikes, this may impede credit supply, and banks may demonstrate less willingness to accommodate jumps in demand for liquidity.[4] Efforts to develop time-varying credit spread adjustments or other mar-

[2]In practice, firms borrow at floating rates and partially hedge their own exposure with derivatives (Chernenko & Faulkender 2011, Kirti 2019). I also examine the use of interest-rate derivatives with the model, by solving for the joint demand for loans and derivatives. There are two ways to rationalize the joint use of floating-rate loans and derivatives in the context of the model. First, this arrangement may help broaden the pool of risk bearing capacity for interest-rate risk; indeed, firms only partially hedge their exposure with derivatives. Second, banks may wish to channel activity to derivatives, where they may have greater market power.

[3]Given the well-documented manipulation of IBORs, I also ask how manipulation affects the value reference rates can provide. Manipulation that adds noise to reference rates reduces welfare gain from the use of reference rates, by weakening the link between reference rates and bank funding costs.

[4]Financial stress at large need not always correspond with spikes in credit risk premia payable by banks. In some periods of financial stress, banks themselves attract funding (Gatev & Strahan 2006, Pennacchi 2006, Cornett, McNutt, Strahan & Tehranian 2011). To the extent this effect is in practice driven by deposit insurance, the credibility of deposit insurance may be relevant.

kets to transfer credit risk, such as swaps linked to the spread between an equivalent of LIBOR and SOFR, may be helpful.

The main contribution of this paper is to present a simple model of the role of reference rates in credit markets. This paper conceptualizes reference interest rates as a tool to facilitate risk sharing. With this perspective, it is natural to link the design of reference rates with the allocation of risk. Duffie, Dworczak & Zhu (2017) show that benchmarks can help alleviate search frictions in over-the-counter markets, and study the conditions under which sellers will choose to publish benchmarks. Duffie & Dworczak (2014) and Coulter, Shapiro & Zimmerman (2017) consider alternative methods of determining reference rates. Eisl, Jankowitsch & Subrahmanyam (2017) and Gandhi, Golez, Jackwerth & Plazzi (2019) discuss approaches to detect and deter manipulation.

The model illustrates that reference rates that incorporate credit risk serve an important purpose in loan markets, by allowing funding risk to be transferred more effectively. Duffie & Stein (2015) point out that multiple types of reference rates may co-exist going forward, with some continuing to incorporate credit risk. With publication of LIBOR slated to cease soon, this will likely only be possible in some jurisdictions. In such cases markets for basis risk will play a particularly important role. Schrimpf & Sushko (2019) provide examples of recent attempts by market participants to construct new benchmarks that do respond to bank credit risk. Kreicher, McCauley & Wooldridge (2014) document that Eurodollar futures, which do incorporate credit risk, grew in popularity during the 1980s, even though Treasury bill futures, which do not incorporate credit risk, were well established by the time Eurodollar futures were introduced. Brousseau, Chailloux & Durré (2013) argue that reference rates should capture bank funding costs across instruments and funding type.

The remainder of this paper is organized as follows. Section 2 provides further background on LIBOR and briefly summarizes recent efforts to replace it. Section 3 details the set up and the three key frictions described above. Sections 4 and 5 analyze the use of reference rates under various institutional arrangements. Section 6 concludes.

2 Background on IBORs and RFRs

LIBOR is now widely used as a reference rate for interest-rate derivatives as well as for loans to firms and households. The name was first used to label an arrangement created to determine interest payments on a $80MN loan to the Shah of Iran in 1969 by a syndicate of banks led by Manufacturer's Hanover (now a part of JP Morgan Chase). Shortly before agreed dates at which the interest rate would be updated, large banks in the syndicate reported their funding costs, an average of which became the new interest rate. As the use of LIBOR became more popular, banks' own funding costs began to be tied LIBOR, providing an incentive for banks to underreport. A more formal version of LIBOR was introduced in 1986 to address this issue, administered by the British Bankers' Association.[5] LIBOR is determined as a trimmed average of submissions from a panel of large banks: each bank submits one number to a calculating agent, which reports an average after discarding some high and low outliers. Each bank is asked to estimate the rate at which it could borrow on the interbank market 'in reasonable market size' in the morning London time.[6] LIBOR is calculated for several currencies and maturities, though the three month dollar LIBOR is the most commonly used rate.

The interbank lending market is an over-the-counter market. When LIBOR and interest-rate swaps were first introduced, interbank markets were a liquid source of bank financing. Since then, large savers have shifted away from bank deposits, forcing banks to fund themselves through certificates of deposits (CDs), or secured financing transactions like repos (Brousseau et al. 2013, BIS 2013). The interbank market, particularly at maturities beyond weeks, is therefore now quite thin. Indeed, even if transactions were collected over a ten day window, only a handful might occur at the three month maturity (Duffie, Skeie & Vickery 2013).

Even as the interbank market has thinned, the volume of assets that reference LIBOR has substantially grown, driven by explosive growth in derivatives volumes. Outstanding notional amounts of interest-rate derivatives have grown to hundreds of trillions of dollars. The Market

[5]This is essentially the present version of LIBOR. See *Business Insider, A Greek Banker on the Early Days of the LIBOR, August 2012* for an account of this first loan. As of February 2014, ICE administers LIBOR.

[6] EURIBOR is subtly different; banks are asked to estimate the rate offered 'by one prime bank to another.'

Participants Group convened by the FSB estimated that more than $300TN of contracts referred to LIBOR, EURIBOR, and other reference rates in 2014.[7] The unfortunate combination of a large volume of transactions related to LIBOR and an opaque and sparse underlying interbank market permitted the recent scandal as banks attempted to manipulate LIBOR. Settlements with regulators show that banks' incentives were driven by direct exposure through interest rate derivatives as well as reputational concerns.[8] The FCA has announced that publication of LIBOR will likely cease after end-2021.

Going forward, a shift towards RFRs has been recommended (Duffie & Stein 2015). Alternative reference rates have been chosen in major jurisdictions in line with this guidance. In the US, SOFR will be used, and is based on a variety of repo funding transactions collateralized by US treasuries. Available on the Federal Reserve Bank of New York's website, SOFR has been officially calculated since April 2018, based on transactions with daily volumes of around $1TN. In the UK, the methodology used to calculate SONIA, which is administered by the Bank of England, was revised in April 2018. The current terms of reference for SONIA are to capture the interest rate on transactions where credit risk in minimal. In the Euro Area, the ECB has published the Euro short-term rate, €STR, since October 2019. The €STR is based on overnight unsecured wholesale funding transactions of Euro Area banks. As Euribor is expected to continue to be published, the EU may be an exanple of a jurisdiction where two reference rates, with one incorporating bank credit risk, persist.

This raises the question of how the change in treatment of credit risk should be addressed. Market participants have extensively discussed this question, attempting to identify an appropriate "benchmark spread adjustment." In the US, the Alternative Reference Rates Committee, ARRC, has made detailed proposals. It has proposed that syndicated loan contracts stipulate that the spread adjustment be chosen by the ARRC itself, by the International Swaps and Deriva-

[7]See the Market Participants Group's Final Report. Notional values of derivatives are not directly comparable to loan volumes: derivatives typically do not involve exchange of principal amounts, and large gross volumes often translate to small net exposures.

[8]See, for example, Barclays' settlement with the FSA. Barclays was the first bank to settle with regulators. Hou & Skeie (2013) provide an extensive timeline of key events.

tives Association – a trade group, or, if necessary, bilaterally between the loan parties. In both the US and UK, the discussion has centered around a time-invariant credit adjustment such as a historical five year median calculated using a fixed historical window. Schrimpf & Sushko (2019) document attempts by market participants to construct reference rates that do incorporate bank credit risk, based on instruments such as unsecured term deposits, commercial paper, and CDs. As they point out, such attempts show that credit risk remains important for a meaningful portion of banks' funding structures.

These issues are explicitly contemplated in private loan contracts. These agreements typically refer to the importance of reference rates that "fairly reflect the cost to ... [l]enders of funding ... [l]oan[s]."[9] Credit risk premia are not always large as a share of total bank funding costs, but are highly uncertain.

3 Framework and frictions

I develop a simple model of maturity transformation incorporating uncertainty regarding future short-term financing costs. In the absence of reference rates, lenders must bear the associated funding risk, as these costs are not contractible. Reference rates are estimates of lenders' current short-term funding costs. Their existence mitigates contractual incompleteness.

As noted earlier, the financial crisis provided one prominent example of funding risk. Figure 1 shows the volume of C&I loans (for US commercial banks), LIBOR, and rates on repo transactions similar to those used to calculate SOFR. Of particular interest is the month (shaded) when the financial crisis was in its acute phase. Suppose that loans backed by short-term financing had to have fixed interest rates. Banks would have to bear the risk that their cost of funding might rise, as it did in this period. In contrast, with floating rates, this risk is transferred to borrowers.[10] Figure 1 shows that reference rates linked to government borrowing costs

[9] See Broadcom and BellRing's agreements, reported to the SEC in Form 8-K filings in May and October 2019 respectively, as two recent examples.

[10] Loan volumes jumped because firms drew down on commitments (Ivashina & Scharfstein 2010). For simplicity, I focus on loans.

only imperfectly capture movements in bank funding costs. Indeed, instead of rising with bank credit risk, repo rates for transactions fell, in part due to flight to safety – an additional source of potential wrong-way risk.

3.1 Set up

I analyze a model with three periods: t_0, t_1, t_2. I will consider a transaction between a single firm and a single bank; as discussed below, the analysis applies more generally. The firm and bank are strategic agents with endogenous demand that responds to prices, while the bank is funded by non-strategic investors. The bank lends L to a firm at t_0 for a project with cash flows $L(1+P)$ at t_2 and no intermediate cash flows. In return, the firm promises to pay $L(1+\mu)$ at t_2. Thus μ is the (fixed) interest rate. The quantity of credit, L, and the interest rate μ, will be determined in equilibrium.

The bank finances itself with a sequence of two short-term loans from investors. It borrows L from investors at t_0, and must pay $L(1+S_0)$ back at t_1 (I make the normalization $S_0 = 0$ to simplify notation). It finances its repayment at t_1 by borrowing L again at t_1, promising to pay $L(1+S_1)$ at t_2. Figure 2 illustrates the timeline. It shows variables per dollar of lending (assuming $L = 1$). Vertical arrows indicate payments and their directions, in the periods that they occur.

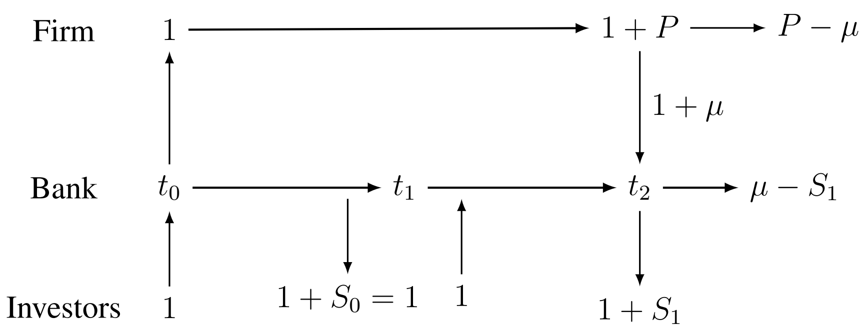

Figure 2: *Timeline (per dollar of lending)*

Project outcomes, P, and short-term funding costs, S_1, are assumed to be Normally distributed (for a generic random variable X, I use the notation $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$, where μ_X and

σ_X^2 are the mean and variance). Uncertainty about S_1, which is not resolved until t_1, reflects potential shifts in both general interest rates and bank credit risk, although I do not explicitly model default. I assume $\mu_P > \mu_S$ so that in expectation the project is worth funding.

The publicly determined reference rate, denoted R_1, will also be Normally distributed. To simplify notation, I assume that the reference rate has mean zero ($\mu_R = 0$). I denote the covariances with short-term funding costs and project outcomes $\mathrm{Cov}(S_1, R_1) = \rho > 0$ and $\mathrm{Cov}(P, R_1) = \pi$ respectively. Clearly the reference rate should be positively correlated with funding costs: this is its basic role. I note assumptions regarding these covariances as used. A baseline case to keep in mind is $\pi = 0$, i.e. that project outcomes are not correlated with the reference rate (and funding costs).

3.2 Frictions

Three key frictions generate meaningful funding risk and a desire to reduce it. First, short-term funding must be used. This assumption reflects the fact that maturity transformation is central to bank intermediation. While in practice some longer term financing is used, banks do not fully match the maturities of their assets and liabilities. A substantial majority of commercial bank liabilities continue to be in the form of interest-bearing deposits. In seminal work, Diamond & Dybvig (1983) model banks as maturity transformers: investing in long-term illiquid assets, while providing liquidity insurance through short-term or demand deposits. Stein (2012) argues that a preference for the safety associated with short-term assets makes it cheaper for banks to fund themselves with shorter term liabilities.[11]

Second, S_1 is not contractible. A contract that referred directly to an individual bank's funding cost would likely be viewed with suspicion. For example, some retail mortgages in the UK are pegged to discretionary rates. During the crisis banks lowered these rates slowly while policy rates were drastically lowered. More recently, banks have at times raised their rates even as policy rates have stayed low.[12] One interpretation is that large corporate borrowers have

[11]It is also possible that short-term financing improves incentives to monitor (Diamond & Rajan 2001).

[12]See, for example, *The Telegraph*, Mortgage borrowers locked into high rates, January 2013. These rates are

sufficient bargaining power to insist on a formal index. As discussed in Section 2, LIBOR began as a syndicate-specific survey for a specific loan. The current formal version was introduced due to concerns with such an arrangement. Reference rates can thus be viewed as the standardized, publicly available, and hence contractible version of bank funding costs.

Finally, I follow Froot et al. (1993) and Froot & Stein (1998), and assume the presence of financial frictions, which lead to effective risk aversion. The crux of the argument is that if today's risky payoffs affect how much can be invested in the following period, when investment opportunities are concave, the profit function inherits this concavity. This concavity creates effective risk aversion. To see the intuition, suppose that following the three periods that are modeled above, the bank has concave investment opportunities $F(I)$. Denote the internal funds that depend on risky payoffs by w. If the firm can raise no external funds, the profit function inherits this concavity: $P(w) = F(w)$. A firm maximizing expected profits when the profit function is concave is effectively risk averse.

Froot et al. (1993) show that even if external financing is available, the profit function is still concave if external financing is costly. Denote the difference between investment and internal funds by $e = I - w$. If this external financing is associated with a convex cost $C(e)$, the profit function continues to be concave.

$$P_{ww} = F_{II} \frac{dI^*(w)}{dw} \propto -F_{II} C_{ee} \tag{1}$$

What financial frictions generate a convex cost of external finance? I follow Froot et al. (1993) and use a version of the costly state verification model of Townsend (1979). More precisely, the output of production denoted by $F(I)$ cannot be used as collateral to borrow (for instance because the project has no liquidation value). Instead all borrowing must be collateralized by risky cash flows y, distributed $g(y)$, generated by existing assets. The only action external financiers can take to force repayment is to liquidate the assets generating these risky cash flows, at a cost. As Appendix A shows, under fairly general conditions this set up generates concavity

known as 'Standard Variable Rates'.

of the profit function.

I use a simplified version of the framework of Froot & Stein (1998) to generate a tractable form for the risk aversion arising from the concavity of the profit function. Suppose the bank has previously chosen some level of capital K, and can currently add θ units of a Normally distributed payoff X to its balance sheet. The funds available to be invested next period are then $w = w_0 + \theta X + K(1-\tau)$. τ is a deadweight cost of cash held on the firm's balance sheet (such as loss of a debt tax shield) so that the unmodeled choice of capital structure is meaningful.

Now the firm's optimal allocation to the payoff X, θ, can be chosen to maximize expected profits. The first order condition provides

$$\begin{aligned}
\frac{dEP(w)}{d\theta} &= E\left(P_w \frac{dw}{d\theta}\right) = \mathrm{Cov}\left(P_w, \frac{dw}{d\theta}\right) + EP_w E\frac{dw}{d\theta} \\
&= EP_{ww}\,\mathrm{Cov}\,(w, X) + EP_w EX \\
&= \theta EP_{ww}\sigma_X^2 + EP_w \mu_X = 0
\end{aligned}$$

where the third equality uses the fact that for x, y normally distributed, $\mathrm{Cov}(f(x), y) = E(f_x)\,\mathrm{Cov}(x, y)$ and that $\frac{dw}{d\theta} = X$. Solving for the optimal allocation

$$\theta^* = \frac{\mu_X}{A\sigma_X^2} \text{ with } A = -\frac{EP_{ww}}{EP_w} \tag{2}$$

the allocation is the same as what would arise from a CARA utility function. The endogenous coefficient A is similar to the standard coefficient of risk aversion, except that the level of curvature of the profit function depends on the earlier level of capital chosen by the bank. I make the argument with reference to banks, but it evidently applies to firms more generally.

In the remainder of this paper I directly view the bank and firm as maximizing CARA utility functions $U(W_0 + \theta X) = -exp(A(W_0 + \theta X))$, with exogenous coefficients of risk aversion

A_B and A_F respectively. The usual first order conditions provide

$$\arg\max_{\theta} EU(W_0 + \theta X) = \frac{\mu_X}{A\sigma_X^2} \tag{3}$$

The relation with Equation 2 is clear. I refer to $A\sigma_X^2$ as the utility cost of risk. This optimal allocation trades off expected return against the utility cost of risk. If there are N participants with CARA preferences, their aggregate behavior is equivalent to a single agent with risk aversion $\frac{\bar{A}}{N}$, where $\bar{A}$ is the harmonic mean of individual risk aversion. In this sense the analysis is not specific to the case of a single bank dealing with a single firm.

4 Basic uses of reference rates

In the absence of a reference rate, a fixed rate loan is the only contract available. As illustrated in Figure 2, per unit borrowed, the firm's payoff is $P - \mu$. Similarly, the bank's payoff is $\mu - S_1$. The equilibrium interest rate, μ^*, equates the quantity of credit demanded by the firm and supplied by the bank based on the optimal allocation in Equation 3.

$$D(\mu^*) = \frac{\mu_P - \mu^*}{A_F\sigma_P^2} = \frac{\mu^* - \mu_S}{A_B\sigma_S^2} = L(\mu^*) \tag{4}$$

For example, the bank might lend \$250 MM to the firm at an interest rate of 500 basis points, or 5%. I refer to $\Phi = A_F\sigma_P^2 + A_B\sigma_S^2$, as the total utility cost of risk. Welfare, the surplus from the contract between banks and firms, is proportional to quantity transacted.

When reference rates are available, there are two ways the lender can reallocate funding risk: floating interest rates and interest-rate derivatives. Both types of contracts are derivatives in the sense that the payments they require are not known in advance, but are determined based on the realized reference rate (R_1). Facilitating the existence of these contracts is the basic purpose of reference rates. Floating rates shift the incidence of funding risk from the bank to the firm. Interest-rate derivatives shift them to the broader market, at an explicit cost.

4.1 Floating rates

I denote the interest rate on floating-rate loans by $R_1 + \nu$. This corresponds to an interest rate of, for example, LIBOR + 300 bps. R_1 is the reference rate, and ν is the fixed premium over the realized reference rate, agreed in advance. Recall that I assume the reference rate has zero mean, though the numerical example reflects the case of a positive mean. I assume that when a reference rate is available, there is no direct cost to writing a floating rate contract, reflecting the public availability of the reference rate. Figure 3 shows the modified timeline.

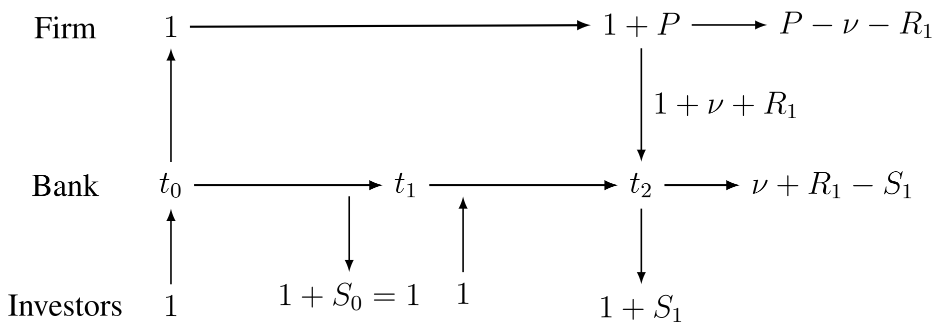

Figure 3: *Timeline for floating rate loans*

As the reference rate has positive covariance with funding costs, the floating rate loan shifts some funding risk from the bank to the firm. However both the bank and the firm now bear risk due to volatility of the reference rate. As long as the reference rate is not too volatile, and $\rho > \frac{\sigma_R^2}{2}$, the bank reduces its risk from $\mathrm{Var}(S_1)$ to $\mathrm{Var}(R_1 - S_1) = \sigma_S^2 + \sigma_R^2 - 2\rho$. This is achieved by increasing risk for the firm from $\mathrm{Var}(P)$ to $\mathrm{Var}(P - R_1) = \sigma_P^2 + \sigma_R^2 - 2\pi$. Again, the baseline case is $\pi = 0$. The new equilibrium interest rate is determined by

$$D'(\nu^*) = \frac{\mu_P - \nu^*}{A_F(\sigma_P^2 + \sigma_R^2 - 2\pi)} = \frac{\nu^* - \mu_S}{A_B(\sigma_S^2 + \sigma_R^2 - 2\rho)} = L'(\nu^*) \tag{5}$$

Relative to Equation 4, the credit supply and demand curves effectively rotate downwards. I refer to $\Phi(R_1) = A_F(\sigma_P^2 + \sigma_R^2 - 2\pi) + A_B(\sigma_S^2 + \sigma_R^2 - 2\rho)$ as the total utility cost of risk when floating rates are used.

Figure 4 illustrates the comparison. D and L are the baseline credit demand and supply curves. D' and L' are the demand and supply curves when a reference rate is introduced. Interest rates ($L' = D'$) fall relative to the baseline case ($L = D$) both because the bank bears less risk and the firm bears more risk.

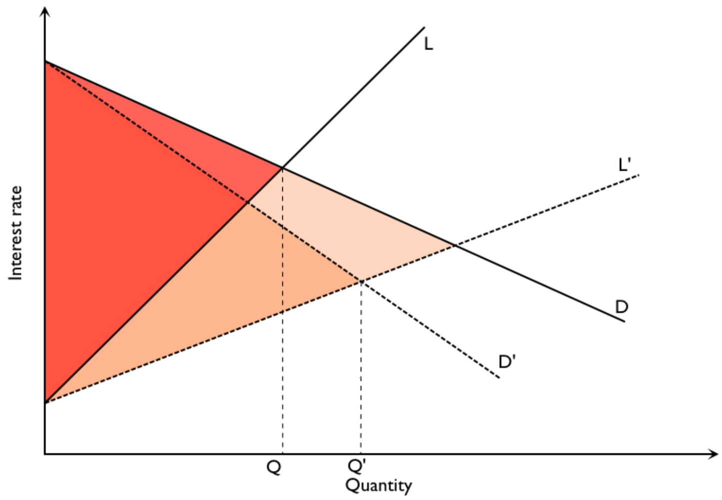

Figure 4: *Fixed and floating rate loans*

Floating rates involve the transfer of risk from the bank to the firm, with the constraint that the reference rate *as a percentage of the loan amount* is transferred, a particularly simple contract. This only improves welfare compared to fixed rates if firms are less risk averse than banks. Proposition 1 formalizes these observations. Ω and $\Omega(R_1)$ denote welfare with a fixed rate and with a floating rate respectively.

Proposition 1. *Floating rates transfer funding risk from the bank to the firm. If $\rho > \frac{\sigma_R^2}{2}$ and $\pi < \frac{\sigma_R^2}{2}$:*

- *Interest rates fall*

$$\mu^* > \nu^* \iff \frac{2\rho - \sigma_R^2}{\sigma_S^2} + \frac{\sigma_R^2 - 2\pi}{\sigma_P^2} > 0 \tag{6}$$

- *However, welfare increases only if the utility benefit of reduced funding risk for banks outweighs the utility cost of increased risk for firms*

$$\Omega(R_1) > \Omega \iff A_B(2\rho - \sigma_R^2) > A_F(\sigma_R^2 - 2\pi) \tag{7}$$

Proof. See Appendix B.1. □

As the potential role for reference rates in this market is to transfer bank funding risk, reference rates that incorporate credit risk are likely to be more useful. Suppose firms' profits are not correlated with reference rates. Welfare with floating rates is then proportional to

$$\frac{1}{A_F(\sigma_P^2 + \sigma_R^2) + A_B(\sigma_S^2 + \sigma_R^2 - 2\rho)} \tag{8}$$

The correlation between funding costs and reference rates, ρ, is likely to be lower for reference rates that not incorporate credit risk, leading to lower welfare. Interestingly, Kreicher et al. (2014) document that Treasury bill futures, which do not incorporate credit risk, predated the introduction of Eurodollar futures, which do. Eurodollar futures grew in popularity over the 1980s even though Treasury futures were already traded in a liquid market. Kreicher et al. (2014) attribute this growth to the fact that hedges based on Treasury bill futures entailed basis risk as well as wrong way risk. Reference rates without credit risk might, of course, be useful when the underlying uncertainty is not about bank funding costs.

4.2 Interest-rate derivatives

Interest-rate swaps provide another way for the bank to reallocate funding risk. I model swaps, denoted $\mathcal{R}_1$, as a contract agreed to at t_0, which (per dollar of notional value) obligates the buyer to pay a fixed rate λ in return for R_1 (which has zero mean), where all payments are made at t_2. The bank can now choose a hedging ratio (the notional value chosen need not be the loan amount) and there may be a direct cost associated with using swaps, to the extent that they are not zero NPV transactions. I begin by taking the unit cost of hedging, λ, as exogenous and defer endogenizing it until Section 5.

How does the use of swaps compare with floating interest rates? For now I consider a return to fixed interest rates, and consider the bank's joint choice of credit supply and hedging with swaps. Denote the fixed rate charged here with κ. The bank's problem is

$$\max_{\theta,\alpha} EU[W_0 + \theta(\kappa - S_1 + \alpha\mathcal{R}_1)] \tag{9}$$

where α is the hedge ratio: the notional value of swaps purchased per unit of lending. $\mathcal{R}_1$ represents both the receipt of R_1 and the fixed payment of λ. With the use of swaps, the bank can construct a contract with payments according to any linear function of the reference rate R_1.[13] Figure 5 shows the modified timeline.

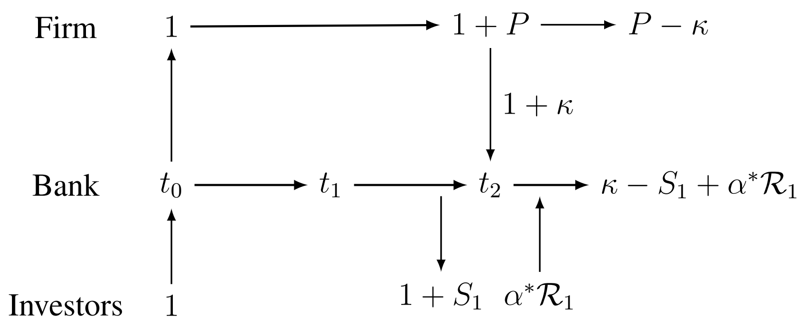

Figure 5: *Timeline when swaps are available*

First order conditions provide

$$L''(\kappa,\lambda) = \theta^* = \frac{\kappa - \mu_S - \lambda\alpha^*(\kappa,\lambda)}{A_B \operatorname{Var}(-S_1 + \alpha^*(\kappa,\lambda)\mathcal{R}_1)} \tag{10}$$

$$\alpha^*(\kappa,\lambda) = \underbrace{\frac{\rho}{\sigma_R^2}}_{\alpha^*} - \frac{\lambda}{A_B\theta^*\sigma_R^2} \tag{11}$$

If the swap is zero NPV ($\lambda = 0$, zero price of risk), the resulting optimal hedging ratio is, with some abuse of notation, $\alpha^* = \alpha^*(\kappa, 0) = \frac{\rho}{\sigma_R^2}$. This optimal hedging ratio is the beta of S_1 with respect to R_1. The bank is willing to supply more credit because it faces less funding risk: $\operatorname{Var}(-S_1 + \alpha^*\mathcal{R}_1) = \sigma_S^2 - \frac{\rho^2}{\sigma_R^2}$ is the residual funding risk after optimal hedging. The optimal level of risk reduction is therefore the beta of funding costs with the reference rate multiplied by their covariance.

[13] If non-linear derivatives on the reference rate were available, it might be optimal to use them. See Appendix C.2 for a discussion of the limited set of situations in which the optimal contract is likely to be linear.

When the derivative is costly, the bank reduces its position size in the derivative: $\alpha^*(\kappa, \lambda) \leq \alpha^*$. However, the bank's demand for loans is the same as if the larger zero-NPV optimal hedge ratio, α^*, is acquired at its market price of λ. As Appendix C.1 shows:

$$L''(\kappa, \lambda) = \frac{\kappa - \mu_S - \lambda\alpha^*(\kappa, \lambda)}{A_B \operatorname{Var}(-S_1 + \alpha^*(\kappa, \lambda)\mathcal{R}_1)} = \frac{\kappa - \mu_S - \lambda\alpha^*}{A_B \operatorname{Var}(-S_1 + \alpha^*\mathcal{R}_1)} = \frac{\kappa - \mu_S - \lambda\frac{\rho}{\sigma_R^2}}{A_B\left(\sigma_S^2 - \frac{\rho^2}{\sigma_R^2}\right)} \quad (12)$$

The bank's demand for loans is therefore independent of the actual quantity of hedging, but of course not independent of the price of hedging.

The equilibrium condition $L''(\kappa) = D(\kappa)$ now allows κ to be determined. Figure 6 illustrates what happens. As before, D and L are the baseline credit demand and supply curves, and D' and L' are the demand and supply curves with floating rates. When derivatives are used for optimal hedging, interest rates ($L'' = D$) are lower than the baseline case ($L = D$) but potentially higher than with reference rates ($L' = D'$).

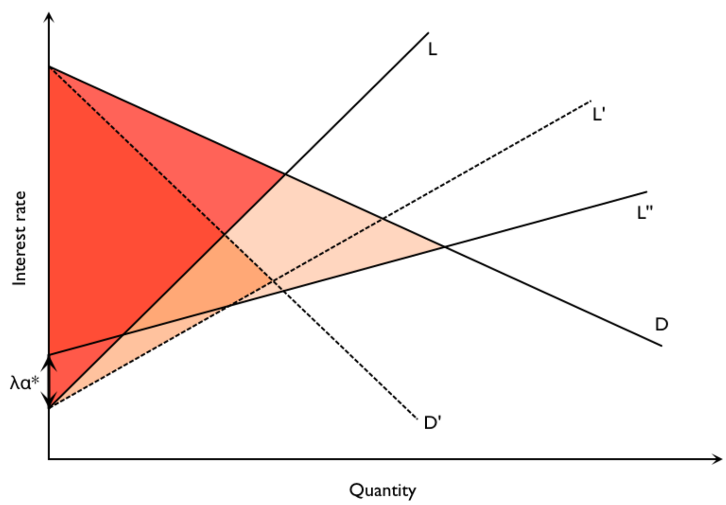

Figure 6: *Floating rate loans and derivatives*

As long as hedging with derivatives is not too expensive, welfare is improved relative to both fixed rates and floating rates. The total cost of hedging, $\lambda\alpha^*$, is best thought of as a fraction of the initial surplus, $\mu_P - \mu_S$ (the interest-rate-intercepts of the original demand and supply curves, D and L in Figure 6). Relative to a fixed rate, as long as this cost is not to high, the reduction in funding risk for the bank lowers interest rates and increases welfare. The effect on interest rates relative to floating rates is ambiguous: optimal hedging means that the risk is lower than with floating rates (strictly, unless $\alpha^* = 1$), pushing rates lower. On the other hand, the firm no longer has to bear risk, which pushes rates higher. Both of these effects raise welfare relative to floating rates. Proposition 2 formalizes these observations. Denote welfare when swaps are available and are zero NPV transactions as $\Omega(\mathcal{R}_1, \lambda\alpha^*)$.

Proposition 2. *If interest rate swaps are zero NPV transactions (*$\lambda = 0$*)*

- *The effect on interest rates relative to floating rates is ambiguous*

$$\nu^* > \kappa^* \iff \frac{\sigma_R^2(\alpha^* - 1)^2}{\sigma_S^2 - \frac{\rho^2}{\sigma_R^2}} > \frac{\sigma_R^2 - 2\pi}{\sigma_P^2} \tag{13}$$

- *Welfare is improved relative to floating rates*

$$\Omega(\mathcal{R}_1, \lambda = 0) > \Omega(R_1) \iff A_B\sigma_R^2(\alpha^* - 1)^2 + A_F(\sigma_R^2 - 2\pi) > 0 \tag{14}$$

If interest rate swaps are costly, welfare is still increased as long as

$$\lambda\alpha^* \leq \bar{C} = (\mu_P - \mu_S)\left(1 - \sqrt{\frac{\Phi(\mathcal{R}_1)}{\Phi(R_1)}}\right) \tag{15}$$

where $\Phi(\cdot)$ *is the total utility cost of risk.*

Proof. See Appendix B.2. $\square$

5 Costly derivatives and hedging by firms

In practice, syndicated loans almost exclusively have floating rates, and it is not uncommon for borrowing firms themselves to use interest-rate derivatives. To understand this behavior it is important to consider how firms would like to use derivatives. I begin by connecting the cost of hedging to aggregate risk tolerance for funding risk when only fixed rate loans are used.

5.1 Cost of hedging

If hedging with swaps were indeed a zero NPV transaction, there would be little reason for floating rates to be used, as these transfer funding risk to effectively risk averse firms. Intuitively, the cost of hedging should be related to overall risk bearing capacity available to bear this risk.

Suppose that dealers or other private capital managers with aggregate risk aversion A_D are willing to enter swap transactions. These dealers provide a supply function for swaps according to Equation 3. The equilibrium cost of hedging, λ^*, is how much these dealers need to be paid to satisfy the bank's hedging demand (hedging demand per unit of credit extended is shown in Equation 11). This condition requires

$$L''(\kappa)\alpha^* - \frac{\lambda^*}{A_B \sigma_R^2} = \frac{\lambda^*}{A_D \sigma_R^2} \tag{16}$$

Recall that $L''(\kappa)$ is the amount of credit supplied, and α^* is the optimal zero-cost hedging ratio.

As discussed in Section 4.2, the bank does not try to shift as much risk to the swap market as implied by the optimal hedging ratio when $\lambda > 0$. Instead, it retains some of the risk, taking the cost of hedging into account. Effectively, this means 'market' risk tolerance combines the risk tolerance of swap dealers and the bank. Define this tolerance, as $T_{D,B}$:

$$T_{D,B} = \frac{1}{A_D} + \frac{1}{A_B} \tag{17}$$

With this notation, Equation 16 can be rearranged to show that the equilibrium cost of hedging is

$$\lambda^* \alpha^* = \frac{1}{T_{D,B}} L''(\kappa) \frac{\rho^2}{\sigma_R^2} \tag{18}$$

Intuitively, the cost of hedging depends on how much risk tolerance there is, how big underlying credit markets are, and how much swaps reduce risk with optimal hedging. Thus Proposition 2 implies that swaps are more effective than floating rates when $T_{D,B}$ is high enough ($\lambda\alpha^* < \bar{C}$).

5.2 Floating rates with hedging by firms

The argument leading to Equation 17 makes clear that aggregate risk tolerance, and therefore the cost of hedging, depends on how broadly funding risk is borne. If firms were to enter swap markets as well, with access to swaps at the market price of λ, they would have hedging demand of their own, similar to the bank demand shown in Equation 11.

$$\beta^*(\kappa, \lambda) = \underbrace{-\frac{\pi}{\sigma_R^2}}_{\beta^*} - \frac{\lambda}{A_F D(\kappa) \sigma_R^2} \tag{19}$$

This is a potential advantage of floating interest rates: they may help draw firms into the market for interest-rate risk. In practice, firms borrow at floating rates, and partially hedge their exposure with interest-rate swaps (Kirti 2019). In principle, even firms with fixed rate liabilities could enter swap markets directly. Although this would lead to similar results, it is perhaps more likely that such activity, as opposed to partial hedging, would be viewed as speculation.

Per unit of credit, the floating rate transfers one unit of exposure to the reference rate from the bank to the firm. This increases the firm's desired hedging ratio by one, and correspondingly decreases the bank's desired hedging ratio by one. Figure 7 shows the modified timeline.

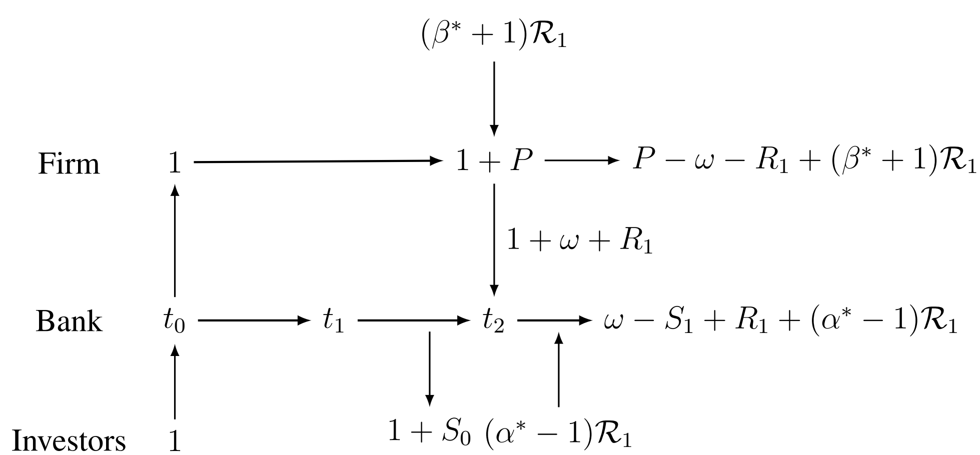

Figure 7: *Timeline with floating rates when both parties can use swaps*

Recall that the baseline case is that $\pi = 0$: that the firm has no initial desire to use reference rates to reduce cash flow risk. In this case $\beta^* = 0$. With floating rates, then, the firm's desired zero-cost hedging ratio is one, and the bank would like $\alpha^* - 1$. In this case the total demand for swaps would be the same as in Equation 16 except that the firm is also willing to bear some risk. The cost of hedging is now determined by

$$L''(\omega)[1 + (\alpha^* - 1)] - \frac{\lambda^*}{A_B \sigma_R^2} - \frac{\lambda^*}{A_F \sigma_R^2} = \frac{\lambda^*}{A_D \sigma_R^2} \tag{20}$$

As the firm absorbs some risk, market risk tolerance is now larger

$$T_{D,B,F} = \frac{1}{A_D} + \frac{1}{A_B} + \frac{1}{A_F} > T_{D,B} \tag{21}$$

This means hedging is cheaper and derivatives are more effective

$$\lambda^* \alpha^* = \frac{1}{T_{D,B,F}} L''(\kappa) \frac{\rho^2}{\sigma_R^2} \tag{22}$$

Note that if reference rates are positively correlated with project outcomes, the exposure the firm obtains through floating rates insures it against cash-flow risks. The firm would then elect to keep even more of the exposure, further reducing the cost of hedging. Proposition 3 formalizes these observations.

Proposition 3. *If $T_D = \infty$ ($\implies \lambda = 0$) and $\pi = 0$:*

- *Fixed rates with bank hedging and floating rates with hedging on both sides are equivalent*
- *Compared to fixed rates without hedging, both lower interest rates in proportion to the reduction in risk $\left(\frac{\rho^2}{\sigma_R^2}\right)$ and increase welfare in proportion to the utility benefit of this reduction in risk $\left(A_B \frac{\rho^2}{\sigma_R^2}\right)$*

If $T_D < \infty$:

- *Welfare is higher with floating rates than with fixed rates*
- *If $\pi \in (0, \rho)$, as firms' optimal hedging ratio is $1 - \frac{\pi}{\sigma_R^2} < 1$, the cost of hedging is decreasing in π and welfare is increasing in π*

Proof. See Appendix B.3. □

5.3 Competition in derivatives markets

Floating rates may help draw firms into the market for interest-rate risk, permitting broader risk sharing and lowering the equilibrium cost of hedging. This is one interpretation consistent with the patterns observed in practice. It is also possible, if derivatives markets are uncompetitive, that universal banks use floating rate loans to channel profitable derivatives activity to their dealer arms. Derivatives markets are in fact highly concentrated: the top six bank holding companies account for about 90 percent of volume for swaps and for derivatives.[14]

Suppose that there are N oligopolistic dealers in the swap market, each symmetrically contributing risk tolerance $\frac{T_D}{N}$. In a symmetric Cournot equilibrium, the cost of of hedging is

$$\lambda' \alpha^* = \frac{1 + \frac{T_D}{N T_{B,F}}}{T_{D,B,F} + \frac{T_D}{N}} L''(\omega) \frac{\rho^2}{\sigma_R^2} > \lambda^* \alpha^* \tag{23}$$

In comparison with the competitive case, shown in Equation 22, the cost of hedging is higher (see Appendix C.3 for details). For instance, if the top six dealers account for even 80% of the total risk tolerance in the market for swaps, Equation 23 implies a cost of hedging 50% higher than the competitive situation described in Equation 22. If the market for swaps is sufficiently concentrated, even with the addition of firms to the set of players that bear funding risk, the cost of hedging may be above the threshold at which they are useful, discussed in Proposition 2.

5.4 Welfare and the cost of manipulation

Contracts linked to reference rates facilitate hedging. Intuitively, their usefulness for this purpose is increased if the covariance between the reference rate and the underlying risk, $\mathrm{Cov}(R_1, S_1) = \rho$, increases. How does manipulation affect reference rates? Portfolio incentives were one im-

[14] The OCC releases quarterly reports on bank trading and derivatives activities. The text refers to the total notional value held by Citigroup, JP Morgan, Goldman Sachs, Bank of America, Morgan Stanley and Wells Fargo as a fraction of the total held by the top 25 banks in Q3 2019.

portant driver of manipulation of LIBOR. These incentives were particularly strong on days when banks were parties to significant volumes of derivative contracts on these days. The resulting manipulation can be thought of as adding pure noise to the reference rate.[15]

I parametrize such manipulation by considering a post-manipulation reference rate

$$\tilde{R}_1 = R_1 + \sqrt{K-1}Z \tag{24}$$

where Z is a normally distributed variable independent of R_1, S, and P, with $\sigma_Z^2 = \sigma_R^2$. K parametrizes the extent of manipulation. Now $\mathrm{Cov}(\tilde{R}_1, S_1) = \mathrm{Cov}(R_1, S_1)$ and $\mathrm{Cov}(\tilde{R}_1, P) = \mathrm{Cov}(R_1, P)$, while $\mathrm{Var}(\tilde{R}_1) = K\sigma_R^2$. I consider the effect of adding a small amount of manipulation, i.e. slightly increasing K from 1.

Intuitively, with zero NPV hedging, such manipulation must reduce welfare as introduces additional, costly, volatility. However, it is less clear what happens when hedging is costly, as institutions will choose to bear more of the underlying risk themselves as contracts linked to reference rates become less useful for hedging. This, in turn, can lower the cost of hedging. Proposition 4 formalizes the idea that this latter effect is not important when the market, in aggregate, is not too risk averse about risk related to the reference rate.

Proposition 4. *For aggregate risk tolerance T high enough, welfare is decreasing in added noise, i.e.*

$$\lim_{T\to\infty} \frac{\partial\Omega(K)}{\partial K} = -\frac{(\mu_P - \mu_S)^2\Phi'(K)}{2\Phi^2(K)} < 0 \tag{25}$$

where $\Phi'(K)$ is the added utility cost of risk participants bear as the reference rate becomes more volatile.

Proof. See Appendix B.4. □

[15] In the model, the main value of reference rates is related to their covariance with funding costs. Reputational incentives may have meant that the level of LIBOR was too low. This could entail broader costs less related to risk sharing.

6 Conclusion

Reference rates are an important part of the financial plumbing that underpins large credit and derivatives markets. The upcoming shift to RFRs will therefore have important implications. This paper focuses on the impact of this shift on credit markets, using a simple model of maturity transformation. RFRs may be appropriate for derivatives markets, where much of the activity may relate to transferring interest-rate risk at large. Their use may, however, lead to unintended consequences in credit markets.

In loan markets, the model highlights that reference rates permit contracts in which borrowers cost of funds is linked tightly to lenders full cost of funds, including credit premia. This provides an important outlet for lenders to manage funding risk. If LIBOR and other IBORs are replaced with RFRs and fixed credit spread adjustments, lenders will have to bear a greater portion of funding risk themselves. This may be problematic at times when lender credit risk spikes. Greater emphasis on credit spread adjustments that track bank lender credit risk and other mechanisms to transfer funding risk may be needed.

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A Effective institutional risk aversion

In this Appendix, I follow Froot et al. (1993) and relate institutional risk aversion to financial frictions. Suppose a bank has risky internal funds w. In the following period it has concave productive opportunities $f(I)$. As internal funds may not be sufficient, the bank would sometimes like to borrow in order to invest more. However, investment and output are not observable to external financiers and cannot be used to collateralize borrowing. Instead, all borrowing must be collateralized by risky cash flows y, distributed $g(y)$, generated by existing assets. This cash flow is observable to external financiers at a cost of c (a cost paid in equilibrium only in the event of default). The financiers are risk neutral and the competitive rate of return they require is normalized to 0. Financiers finance the gap between investment and internal funds, $I - w$, in return for a non-state contingent repayment with face value D in the next period.

The bank maximizes the value of output and the portion of y it retains, by choosing I and D, subject to the lender's IR constraint:

$$P(w) = f(I) + \int_D^{\infty} (y - D)g(y)dy + \lambda \left(\int_{-\infty}^{D} (y - c)g(y)dy + \int_D^{\infty} Dg(y)dy - (I - w) \right) \tag{26}$$

The first order conditions are

$$\frac{\partial P}{\partial D} = (\lambda - 1)(1 - G(D)) - \lambda c g(D) = 0 \tag{27}$$

$$\frac{\partial P}{\partial I} = f_I - \lambda = 0 \tag{28}$$

Bankruptcy costs (the need for financiers to expend c in the event of default) generate underinvestment

$$f_I = \frac{1 - G(D)}{1 - G(D) - cg(D)} > 1 \tag{29}$$

assuming an interior solution (i.e. $1 - G(D) - cg(D) > 0$) and positive monitoring costs.

The first order condition with respect to w implies that $P_w = \lambda$, while Equation 28 implies that $\lambda = f_I$. The concavity of the profit function is therefore determined by

$$P_{ww} = f_{II} \frac{dI^*}{dw} \tag{30}$$

This is the same expression as Equation 1. In order for the profit function to be concave, then, investment must respond positively to the level of internal funds so that $P_{ww} < 0$. This can be guaranteed when g has an increasing hazard rate.

Begin by combining Equations 27 and 28 and totally differentiating with respect to w:

$$\begin{aligned} (f_I(I^*(w)) - 1)(1 - G(D(w))) &= f_I(I^*(w))cg(D(w)) \\ \implies f_{II} \frac{dI^*}{dw}(1 - G(D)) - G'(D)D'(w)(f_I - 1) &= f_{II} \frac{dI^*}{dw} cg(D) + f_I cg'(D)D'(w) \end{aligned} \tag{31}$$

Next, I solve for $D'(w)$, by totally differentiating the lender's IR constraint, which determines how D responds to changes in w:

$$\begin{aligned} \int_{-\infty}^{D(w)} (y - c)g(y)dy + \int_{D(w)}^{\infty} D(w)g(y)dy &= I^*(w) - w \\ \implies D'(w) &= \frac{\frac{dI^*}{dw} - 1}{1 - G(D) - cg(D)} \end{aligned} \tag{32}$$

Substituting Equations 32 and 29 into Equation 31 provides an expression for $\frac{dI^*}{dw}$ in terms of f'', c, and g:

$$\begin{aligned} \frac{dI^*}{dw} &= \frac{1}{1 - f_{II}\Gamma} \\ \text{where } \Gamma &= \frac{1}{c} \frac{(1 - G(D) - cg(D))^3}{g(D)G'(D) + g'(D)(1 - G(D))} \end{aligned}$$

As f is concave and I have already assumed $1 - G(D) - cg(D) > 0$, $\frac{dI^*}{dw}$ has the same sign as

$$g(D)G'(D) + g'(D)(1 - G(D)) \propto \frac{d}{dD} \frac{g(D)}{1 - G(D)}$$

For concavity of the profit function it is therefore sufficient for the hazard rate of g to be strictly increasing. It can also be shown that the same condition generates a convex cost function (where the cost is the additional deadweight cost arising from external finance). This condition is satisfied, for example, for the Normal and Uniform distributions.

B Proofs

B.1 Proof of Proposition 1

Let $X_D = A_F \sigma_P^2$ and $X_S = A_B \sigma_S^2$ be the utility costs of risk on the demand and supply sides of the market when a fixed rate is used. Then the equilibrium price is determined as

$$\frac{\mu_P - \mu^*}{X_D} = \frac{\mu^* - \mu_S}{X_S} \tag{33}$$

$$\implies \mu^* = \frac{\mu_S X_D + \mu_P X_S}{X_D + X_S} \tag{34}$$

Denote welfare when a fixed rate is used by Ω. Welfare depends on the surplus from lending, $\mu_P - \mu_S$:

$$\Omega = \frac{1}{2} \frac{(\mu_P - \mu_S)^2}{X_D + X_S} \tag{35}$$

The introduction of a floating rate changes the utility costs of risk to

$$Y_D = A_F \left(\sigma_P^2 + \sigma_R^2 - 2\pi\right)$$

$$Y_S = A_B \left(\sigma_S^2 + \sigma_R^2 - 2\rho\right)$$

The expressions for ν^* and $\Omega(R_1)$ (welfare when a floating rate is used) are similar to Equations 34 and 35. Rearrangements show that the change in interest rates is

$$\mu^* - \nu^* = \frac{(\mu_P - \mu_S)(Y_D X_S - X_D Y_S)}{(X_D + X_S)(Y_D + Y_S)} \tag{36}$$

This can be written as

$$\mathrm{Sgn}(\mu^* - \nu^*) = \mathrm{Sgn}\left(\frac{X_S - Y_S}{X_S} - \frac{X_D - Y_D}{X_D}\right) \tag{37}$$

Similarly the sign of the welfare difference is

$$\mathrm{Sgn}\left(\Omega(R_1) - \Omega\right) = \mathrm{Sgn}((X_D - Y_D) + (X_S - Y_S)) \tag{38}$$

This proves Proposition 1.

B.2 Proof of Proposition 2

Equations 13 and 14 follow from the same argument used to prove Proposition 1. This leaves Equation 15. Denote welfare when swaps are used and their exogenous cost is λ by $\Omega(\mathcal{R}_1, \lambda\alpha^*)$. I have already established that $\Omega(\mathcal{R}_1, 0) > \Omega(R_1)$. Let Z_D and Z_S be the utility costs of risk when swaps are used. The threshold cost of optimal hedging can be found by equating these levels of welfare

$$\begin{aligned} \Omega(\mathcal{R}_1, \bar{C}) &= \Omega(R_1) \\ \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{Y_D + Y_S} &= \frac{1}{2}\frac{(\mu_P - \mu_S - \bar{C})^2}{Z_D + Z_S} \end{aligned}$$

From this quadratic equation I select the smaller root

$$\bar{C} = (\mu_P - \mu_S)\left(1 - \sqrt{\frac{Z_D + Z_S}{Y_D + Y_S}}\right)$$

as the maximal cost must be smaller than the surplus $\mu_P - \mu_S$. The total utility costs of risk are $\Phi(\mathcal{R}_1) = Z_D + Z_S$ and $\Phi(R_1) = Y_D + Y_S$.

This proves Proposition 2.

B.3 Proof of Proposition 3

The discussion prior to Proposition 3 explains why when hedging is a zero NPV transaction, fixed rates with the bank hedging with swaps and floating rates with both the firm and the bank hedging with swaps are equivalent. For this part I assume that $\pi = 0$. The same reasoning used in the proof of Proposition 1 explains the interest rate and welfare differences relative to fixed rates.

Similarly, the discussion before Proposition 3 establishes that the cost of hedging is lower with floating rates when firms also hedge, as risk tolerance is higher. Welfare, as a function of risk tolerance in the market for swaps, is

$$\Omega(T) = \frac{1}{2}\frac{(\mu_P - \mu_S - \lambda(T)\alpha^*)^2}{\Phi(\rho)} \tag{39}$$

where $\Phi(\rho)$ takes into account that the bank behaves as if it has reduced its risk by $\rho\alpha^*$ (continue to assume $\pi = 0$ for now). Substituting for the cost of hedging from Equation 18, I find

$$\begin{aligned} \lambda(T)\alpha^* &= \frac{\mu_P - \mu_S}{T\Phi(\rho)}\rho\alpha^* \\ \implies \Omega(T) &= \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(\rho)}\left(1 - \frac{1}{T\Phi(\rho)}\frac{\rho^2}{\sigma_R^2}\right)^2 \end{aligned} \tag{40}$$

When $\pi \neq 0$, this expression becomes[16]

$$\Omega(T) = \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(\rho, \pi)}\left(1 - \frac{1}{T\Phi(\rho, \pi)}\frac{(\rho - \pi)^2}{\sigma_R^2}\right)^2 \tag{41}$$

For $\pi \in (0, \rho)$, the cost of hedging is decreasing in π and welfare is increasing in π.

This proves Proposition 3.

[16] Note that the corresponding version of Equation 18 then provides an expression for $\lambda(\alpha^* + \beta^*)$.

B.4 Proof of Proposition 4

Welfare as a function of the level of manipulation K is

$$\Omega(K) = \frac{1}{2}\frac{(\mu_P - \mu_S)^2}{\Phi(K)}\left(1 - \frac{1}{T\Phi(K)}\frac{(\rho-\pi)^2}{K\sigma_R^2}\right)^2 \tag{42}$$

The effect of increasing K is

$$\frac{\partial\Omega(K)}{\partial K} = \left(1 - \frac{1}{T\Phi(K)}\frac{(\rho-\pi)^2}{K\sigma_R^2}\right)\left(\frac{1}{T}\frac{(\mu_P-\mu_S)^2(\rho-\pi)^2}{2\Phi^2(K)K\sigma_R^2}\underbrace{\left(\frac{2}{K}+\frac{3\Phi'(K)}{\Phi(K)}\right)}_{>0} - \underbrace{\frac{(\mu_P-\mu_S)^2\Phi'(K)}{2\Phi^2(K)}}_{>0}\right) \tag{43}$$

The two marked terms are positive because $\Phi'(K) > 0$: both lenders and borrowers bear more risk as K increases. For example, the optimal level of risk reduction for the bank is $\frac{\rho^2}{K\sigma_R^2}$. The effect of added risk when hedging is zero NPV is a clear negative effect when T is sufficiently large

$$\lim_{T\to\infty}\frac{\partial\Omega(K)}{\partial K} = -\frac{(\mu_P-\mu_S)^2\Phi'(K)}{2\Phi^2(K)} < 0 \tag{44}$$

This proves Proposition 4.

C Details

C.1 Optimal loan supply with costly hedging

Begin by simplifying the notation: let $H = \kappa - \mu_S$, $\sigma_R^2 = 1$ and $\sigma_S^2 = \sigma^2$. I want to show

$$\frac{\kappa - \mu_S - \lambda\alpha^*(\kappa,\lambda)}{A_B\,\mathrm{Var}(-S_1 + \alpha^*(\kappa,\lambda)\mathcal{R}_1)} = \frac{\kappa - \mu_S - \lambda\alpha^*}{A_B\,\mathrm{Var}(-S_1+\alpha^*\mathcal{R}_1)}$$

$$\iff \frac{H - \lambda\left(\rho - \frac{\lambda}{A\theta}\right)}{A(\sigma^2 + \left(\rho - \frac{\lambda}{A\theta}\right)^2 - 2\left(\rho-\frac{\lambda}{A\theta}\right)\rho)} = \frac{H-\lambda\rho}{A(\sigma^2-\rho^2)} = \theta$$

The second equality can be verified from

$$\frac{H - \lambda\left(\rho - \frac{\lambda}{A\theta}\right)}{A(\sigma^2 + \left(\rho - \frac{\lambda}{A\theta}\right)^2 - 2\left(\rho - \frac{\lambda}{A\theta}\right)\rho)} = \frac{A\theta(H - \lambda\rho) - \lambda^2}{A^2\theta(\sigma^2 - \rho^2) + \frac{\lambda^2}{\theta}}$$

$$= \frac{A\theta(H - \lambda\rho)}{A^2\theta(\sigma^2 - \rho^2)} \frac{\overbrace{A^2\theta(\sigma^2 - \rho^2)}^{=X}}{\underbrace{A^2\theta(\sigma^2 - \rho^2) + \frac{\lambda^2}{\theta}}_{=Y}} + \frac{\lambda^2}{\underbrace{A^2\theta(\sigma^2 - \rho^2) + \frac{\lambda^2}{\theta}}_{=Y}} = \theta$$

where the final equality follows from $\theta(Y - X) = \lambda^2$.

C.2 Optimal contracting

In general the optimal contract need not be linear. Consider a simplified problem: suppose P, S are functions of a random variable R. In the more general problem I analyze in this paper, P, S_1 and R_1 are jointly distributed, but this simple version provides useful intuition. Consider a standard optimal risk sharing problem. What is the optimal function for the payment from the firm to the bank $f(R)$ subject to a participation constraint for the bank? Suppose R has pdf $g(R)$. The Lagrangian for this problem is

$$\mathcal{L} = \max_{f(R)} \int U^F[P(R) - f(R)]g(R)dR + \lambda\left(\bar{U} - \int U^B[f(R) - S(R)]g(R)dR\right) \quad (45)$$

Pointwise maximization provides the Borsch risk sharing rule, where I write the marginal utilities as functions of the resulting wealth.

$$-\frac{U_W^F(P(R) - f(R))}{U_W^B(f(R) - S(R))} = \lambda \quad (46)$$

Implicit differentiation and rearrangement provides

$$\left[\underbrace{-\frac{U_{WW}^F}{U_W^F}}_{A_F} \left(\frac{df^*(R)}{dR} - \frac{dP}{dR} \right) \right] + \left[\underbrace{-\frac{U_{WW}^B}{U_W^B}}_{A_B} \left(\frac{df^*(R)}{dR} - \frac{dS}{dR} \right) \right] = 0 \tag{47}$$

and therefore

$$\frac{df^*(R)}{dR} = \frac{A_F \frac{dP}{dR} + A_B \frac{dS}{dR}}{A_F + A_B} \tag{48}$$

Thus f should be linear only if P and S are linear functions of R.

C.3 Competition in derivatives markets

Equation 20 can be rearranged to find an inverse demand function from the perspective of dealers, as a function of Q, the total demand for swaps that all N dealers choose.

$$\lambda(Q) = (L''(\omega)\alpha^* - Q)T_{B,F}\sigma_R^2 \tag{49}$$

$T_{B,F}$ is the combined risk tolerance of banks and firms. An individual dealer's problem is then to maximize production, q_i, holding $r = \sum_{j \neq i} q_j$ fixed. This objective can be written as

$$\max_{q_i} \; q_i \lambda(q_i + r) - \frac{1}{2} A_i q_i^2 \sigma_R^2 \tag{50}$$

The dealer takes into account how its demand decision will affect its return for accepting risk as well as the utility cost of bearing this risk. Recall that I have assumed $A_i = \frac{N}{T_D}$. The first order condition provides

$$q_i = \frac{T_{B,F}(L''(\omega)\alpha^* - r)}{2T_{B,F} + A_i} \tag{51}$$

In a symmetric equilibrium it must be the case that $r = (N-1)q$. Substituting this in, each dealer demands

$$q^* = \frac{T_{B,F} L''(\omega)\alpha^*}{(N+1)T_{B,F} + A_i} \tag{52}$$

The equilibrium cost of hedging, as in Equation 23, is

$$\lambda(Nq^*)\alpha^* = \frac{1+\frac{T_D}{NT_{B,F}}}{T_{D,B,F}+\frac{T_D}{N}} L''(\omega)\frac{\rho^2}{\sigma_R^2} \tag{53}$$

As N goes to infinity, this approaches the competitive cost of hedging, $\lambda^*\alpha^*$, shown in Equation 22. To see that it is always greater, note that

$$\frac{1+\frac{T_D}{NT_{B,F}}}{T_{D,B,F}+\frac{T_D}{N}} = \frac{1}{T_{D,B,F}}\left(1+\underbrace{\frac{T_D^2}{T_{B,F}(T_D+NT_{D,B,F})}}_{>0}\right) \tag{54}$$