Demographics and the Real Exchange Rate

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Abstract

We develop a two-country, two-sector overlapping generations model to examine the effect of population aging on the real exchange rate (RER). While an older population raises the relative demand for nontradables (a feature of structural transformation) putting upward pressure on relative prices thus appreciating the RER, it also implies a lower real interest rate (distinctive of secular stagnation) that dampens the elderly nontradables consumption and thus mitigates the RER appreciation. We quantify a general equilibrium effect of 0.1% RER appreciation following a rise by 1% in the relative old dependency ratio in line with our empirical estimates.

JEL codes: F31, F41, J11.

Keywords: population aging, real exchange rates, life-cycle saving, consumption composition

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1 Introduction

The world's population is aging: the relative number of elderly is increasing in all advanced economies. According to recent research, aging is an important driver of two key phenomena that are characterizing the economic development: the systematic reallocation of economic activity to the services sector ("structural transformation", see Cravino et al. (2019)) and a context of persistently low real interest rates ("secular stagnation", see Eggertsson et al. (2019)). We provide the first general equilibrium framework that connects these two phenomena in an open-economy environment to analyze the impact of aging on the real exchange rate (RER).

Aging can matter for the determination of the relative price of nontradables and hence of the RER for two main facts. First, while all advanced economies are aging, extent and timing differ across regions. This is well documented by the UN (2017) projections. The old dependency ratio, i.e. the ratio of the elderly (aged 65+) to the working-age population (aged 15-64), was about 0.2 for the United States and Europe, and about 0.1 for Japan in the 1980s and it is now projected to rise to about 0.55 in Europe, 0.35 in the United States, and 0.7 in Japan by 2050.[1] Second, what we consume changes with age. By means of survey data, we document that older consumers devote a considerably higher share of their expenditures to nontradable goods and services. A household aged 80 devotes about 15 percentage points more of its expenditure to nontradables as compared to a household aged less than 60.[2]

As population ages the economy experiences a shift of aggregate demand in favor of nontradables with ensuing adjustment of the supply side (structural transformation). Our framework incorporates this demand-driven sectoral shift towards nontradables occurring with aging and it is such that a country aging more than its trading partners faces a higher increase of the relative price of nontradables and so a RER appreciation. Our Europe-US calibrated model predicts about 11% appreciation between 1960 and 2050 in partial equilibrium. The appreciation reduces to about 5% in a general equilibrium environment of declining real interest rates

[1] See Figure 16 in appendix for a representation of demographic patterns.

[2] See Figure 1. This pattern is robust across EU and US over time, different degrees of detail in the classification of the consumption categories into tradables and nontradables, year and family-composition effects. The categories responsible for this shift are mostly health-care and housing.

(secular stagnation) induced by aging. As a result, a general equilibrium approach is needed to capture the role of demographics on the RER.

As motivating contribution we first test empirically the link between demographics and the long-run RER using cointegration techniques on panel data, extending the work of Ricci et al. (2013). We find that a 1% increase in the old-dependency ratio of a country relative to its trading partners is associated with an appreciation of its real effective exchange rate (REER) of 0.29%. This number, in line with the existing literature (Groneck and Kaufmann, 2017)[3] and corresponding to our model estimates under certain calibrations of the pension systems, entails that demographics explain about 15% of the mean absolute deviation of REER implied by fundamentals in the sample of advanced economies. This empirical motivating evidence suggests an appreciating force of relative aging at least as important as the Balassa-Samuelson effect[4] for a large number of countries over the long-run consistent with the structural mechanisms embedded in our model.

We develop a two-country overlapping generations (OLG) model with tradable and nontradable goods. On top of factors standard in the literature such as consumption smoothing, different sectoral productivities and factor intensities, and social security, we model a framework that accommodates demand side effects in general equilibrium. We build on the multi-country large-scale OLG models of the world economy of Domeij and Flodén (2006) and Krueger and Ludwig (2007), adding two sectors and age-dependent sectoral consumption shares. We further allow for persistent wage differentials between sectors via imperfect substitutability of working hours. We calibrate the model for Europe (EU) and the United States (US) taking the UN (2017) demographic data as exogenous variation to quantify the effect of aging on the RER. In the model, the RER is an increasing function of the relative price of nontradables only. So a country experiencing a higher increase of the relative price of nontradables experiences a RER appreciation.

Our model is rich enough to allow for an impact of demographics on the relative prices through different channels among which demand composition and saving

[3]Compared to our empirical analysis, Groneck and Kaufmann (2017) have different samples and data sources, and their dependent variable is the relative price of nontradables instead of the REER.
[4]Cf. Bordo et al. (2017).

patterns are the dominant ones. First, thanks to age-specific consumption shares, aging implies higher aggregate relative demand for nontradables which induces a higher relative demand for nontradable labor. To attract labor in this sector, in the presence of imperfect substitutability of working hours, the relative hourly wage has to increase permanently in the nontradable sector. This translates into a higher relative price of nontradables. Second, even in the absence of differences in the demand composition across ages, there is a savings channel. Discounting higher survival probabilities, individuals in an aging economy are willing to save more. For given real interest rate, this translates into higher saving rates and in turn higher expected consumption of the elderly. The implied higher consumption in nontradables needs to be met domestically, leading (in presence of imperfect labor mobility) to an increase of the relative price of nontradables. The story would end here if we were analyzing two small-open-economies that take a fixed real interest rate as given.

In addition, there are general equilibrium forces that play an important role in mitigating the two channels above. Faced with the need of financing a longer life individuals react to increases in the life expectancy with a higher willingness to save at the same time in which labor-input becomes scarcer. In general equilibrium, both factors contribute to lowering the real return on savings by making capital relatively more abundant than labor and depressing the marginal product of capital. A lowered real interest rate dampens the increase in the consumption of the elderly (biased for nontradables) and the associated increase in the relative prices of nontradables.[5] The presence of a pay-as-you-go pension system influences the savings behavior crowding-out productive capital as the labor tax rate rises to finance the enlarging pool of retirees. Therefore, if aging economies adopt different pension systems over time there might be an impact on their RER. Whether the partial equilibrium effects prevail on the general equilibrium ones is ultimately a quantitative question that we address in this paper.

Our quantitative model results highlight that due to differences in the demo-

[5]The downward impact of lower fertility and mortality rates on the real return on capital is a standard result of OLG models in the spirit of Auerbach and Kotlikoff (1987) like ours. For a further discussion of the channels involved and quantification see e.g. Krueger and Ludwig (2007), Carvalho et al. (2016), Gagnon et al. (2016).

graphic change between EU and US the deviation of the RER from its initial steady state value (1960) in the long-run (2050) ranges from about 3% to 11%. The range depends on whether or not one allows for long-run change in the growth rate of the relative labor productivity of the tradable sector and general equilibrium effects (particularly the downward pressure on the real interest rate due to aging) as well as country-specific characteristics. These numbers represent a RER appreciation for the area that is projected to age more, EU. In the baseline, our quantitative model estimates that a 1% increase in the old-dependency ratio (the fraction of retirees to workers) of a EU relative to US (its only trading partner in the model) appreciates its RER by about 0.2% in partial and 0.1% in general equilibrium.

This paper contributes to a vast literature on the role of demographics in shaping the macroeconomy (see Aksoy et al. (2019), Cooley and Henriksen (2018) and the references therein) and to a likewise vast literature on the long-run determinants of the RER. With foundations dating back at least as far as the Balassa-Samuelson model,[6] cross-country differentials in the relative price of nontradable goods and services are understood to be important, if not the sole, determinants of the RER in the long-run.[7] Generally, cross-country differentials in a broad set of fundamentals are advocated as determinants of the long-run relative price evolution, where demographics have been considered only relatively recently.[8] The role of demographics on relative prices through demand channels has been approached in partial equilibrium small open economy models by the existing literature (see Braude (2000), Rose et al. (2009), and Groneck and Kaufmann (2017)). We extend this literature suggesting a comprehensive framework for describing and quantifying the long-run RER dynamics as an outcome of structural shifts to nontradables due to demand composition forces in a setting with lowering interest rates. Doing this, our paper shows that the appreciation of RER that the literature identifies in partial equilibrium is significantly dampened in general equilibrium. In seeing demographics as a deep underlying phenomenon behind structural shifts and persistent low interest

[6] See Balassa (1964) and Samuleson (1964). Some studies recognize the contribution of Harrod (1933) and call it the Harrod-Balassa-Samuelson model.

[7] See Berka et al. (2018) for a recent assessment.

[8] In a review of the literature, Hassan et al. (2011) concluded that *"the relationship between age structure and the real exchange rate is neither theoretically nor empirically well established"*.

rates we contribute to the literature of both secular stagnation and structural transformation. In particular, only few papers in the literature focus on the effect of demographics on the natural interest rate in open economy (see Eggertsson et al. (2016), Coeurdacier et al. (2015)) to which we contribute by targeting specifically the long-term behavior of the RER as well as the role of heterogeneity in the consumption basket across ages.

The rest of the paper is organized as follows. Section 2 provides evidence on the effect of demographics on the long-term RER in the data using panel cointegration techniques. Section 3 provides empirical evidence on the age pattern of sectoral consumption. Section 4 describes the model and the main mechanisms that cause demographic change to impact the RER. Section 5 discusses the calibration. Section 6 presents the quantitative results from the calibrated model for the long-run. Section 7 does a sensitivity analysis. Section 8 compares the theoretical to the empirical results. Section 9 concludes the paper.

2 Motivating evidence on the role of demographics on the real exchange rate

We first test empirically whether aging matters as a fundamental in explaining cross-country long-run differences in the real effective exchange rate (REER). We will then exploit the model to illustrate the mechanism at play. Our theoretical hypothesis is that countries aging more are those whose REER appreciates more.

We use a dataset covering over 90% of the world trade compiled by Ricci et al., 2013 (RMFL henceforth) including refined measures of the fundamentals that might be significantly affecting the REER.[9] We add one explanatory variable that captures the demographic change to the set of fundamentals in this dataset to test whether a panel cointegrating relationship exists.

[9]To our knowledge, it is the most comprehensive dataset for this purpose.

2.1 Data, variables and econometric methodology

We use the dataset constructed by RMFL labeled as "large sample" that comprises 45 countries (both advanced and emerging economies) for 25 years: 1980-2004.[10]

The dependent variable is the Consumer-Price-Index (CPI) based REER computed from the IMF's International Financial Statistics database where an *increase* means *appreciation*. The set of fundamentals analyzed by RMFL comprises: (i) terms of trade based on commodity prices; (ii) net foreign assets relative to trade volume; (iii) government consumption relative to GDP; (iv) labor productivity growth between tradables and nontradables; (v) an index of trade restriction; (vi) an index that captures how much prices are 'administrated' and thus controlled.[11]

We include an additional fundamental to capture demographics. To test whether countries aging more face a REER appreciation, we use the old-dependency ratio defined as the number of people aged 65 or more over the number of people aged between 20 and 64 (in line with Groneck and Kaufmann (2017)). To be consistent with the dependent variable and the other explanatory fundamentals in the dataset provided by RMFL (which are measured *relatively* to the trading partners), our explanatory variable is the trade weighted relative old dependency ratio computed from the United Nations *World Population Prospects: The 2015 Revision* using bilateral trade weights provided by the Bank of International Settlement – see Appendix C.1.1 for details. We denote it as $odrw$ and we use its natural logarithm in the regressions.

We use the same econometric methodology of RMFL. First we provide evidence that the variables exhibit unit root behaviour and that there is panel cointegration among our variables.[12] We then perform the main estimation by pooling the data

[10] Because of missing data on demographics and trade weights, we dropped 3 countries (Taiwan, Pakistan, Morocco) from the original "large sample". Therefore, the resulting sample has 45 countries for the period 1980-2004 which we label as "demographic sample".

[11] Details on how the dataset was built and the economic intuition that links these fundamentals to REER are provided in RMFL. We provide a summary description of the overall dataset and single variables in Appendix C.1.1.

[12] This is a precondition for any further cointegration analysis. In Appendix C.1.2 (Tables 8 and 9) we provide evidence that all variables are non-stationary according Pesaran (2007)'s panel unit root test statistic accounting for cross-sectional dependence and that there exists panel cointegration among our variables according to the Pedroni (1999)'s seven panel cointegration tests.

given the limited length of the sample period (25 years). The main methodology applied is the dynamic ordinary least squares (DOLS) developed by Stock and Watson (1993) correcting standard errors with Newey-West method. The DOLS estimating equation applied to a panel of 45 countries indexed by i over years t reads:

$$\log(REER)_{i,t} = \alpha_i + X_{i,t}\beta + \sum_{s=-p}^{s=p} \Delta X_{i,t+s}\gamma_s + u_{i,t} \tag{2.1}$$

where, exactly as in RMFL, α_i is a vector of country fixed effects, β is the vector of long-run cointegrating coefficients, Δ is the first-difference operator, γs are the $2p+1$ vectors of coefficients of leads and lags of changes in the determinants, and $u_{i,t}$ is the residual term. $X_{i,t}$ is the vector of all the explanatory variables in RMFL and our new explanatory variable, the log of old-dependency ratio relative to the trading partners. Notice that country-specific intercepts, α_is, are necessary as the dependent variable is an index not comparable across countries.

As a robustness, we also apply a different estimating methodology, not used by RMFL, that accounts for issues of cross-sectional dependence. We employ the Common Correlated Effects Pooled (CCEP) estimator developed by Pesaran (2006). Following Pesaran (2006) the common correlated effect (CCE) model to estimate is:

$$\begin{aligned} \log(REER_{i,t}) &= \alpha_i + X_{i,t}\beta_i + e_{i,t} \\ e_{i,t} &= F_t\delta_i + \varepsilon_{i,t} \end{aligned} \tag{2.2}$$

where $\beta_i = \beta + \nu_i, \nu_i \sim IID(0, \Omega_\nu)$, F_t is a vector of unobserved common factors (events that influence all countries at the same time) to which countries are allowed to react differently (δ_i) and $\varepsilon_{i,t}$ are the country-specific errors assumed to be independently distributed of $X_{i,t}$. Notice that since our model is the CCE pooled, the idiosyncratic coefficients on the explanatory variables are constrained to be homogeneous across-countries $\beta_i = \beta, \forall i$. Pesaran (2007) shows that (2.2) can be consistently estimated by approximating F_t with cross section means of the dependent and independent variables under strict exogeneity. Therefore our estimating

equation under CCEP will be:

$$\log(REER_{i,t}) \quad = \quad \alpha_i + X_{i,t}\beta_i + \overline{Z}_{i,t}\lambda_i + \zeta_{i,t} \tag{2.3}$$

where $\overline{Z} = (\overline{\log(REER_{i,t})}, \overline{X}_{i,t})$ denote the cross country means. Once we run the regression to have the CCEP estimator we check whether the estimated model accurately captures the cross-sectional dependence using Pesaran (2015)'s test of the null hypothesis that the error terms are weakly cross-sectionally dependent.

2.2 Estimation results

Table 1 reports the main empirical estimates.

Column (1) replicates Table 4 (2nd column) in RMFL which is obtained using the "large sample" (48 countries). We first confirm that the main results in RMFL are unaffected when we use our 45-countries "demographic sample" (column (2)).[13] Then, with the same DOLS methodology, we add our new regressor, the log of old dependency ratio relative to trading partners. One can see in column (4) that the estimates on the effect of the fundamentals used by RMFL remain largely unaffected even if both the trade restriction index and the variable for price controls are now statistically insignificant.

Our new variable of interest has the expected positive sign with a p-value of 1.4%. An increase of 1% of the old dependency ratio of a country relative to its trading partners is associated with an appreciation of its REER by 0.29%.

In column (3) and (5) we repeat the regressions using the CCEP methodology of equation (2.3) which captures the cross-sectional dependence as the Pesaran (2015)'s CD test strongly rejects the null hypothesis of weakly cross-sectional dependent error term in both cases (see note of Table 1). Using the CCEP methodology the estimates are broadly in line with those obtained with DOLS even if the estimated coefficients on log commodity terms of trade, net foreign assets to trade, government consumption to GDP and price controls are smaller. Importantly, column (5) shows that the CCEP methodology confirms the magnitude of the estimate

[13] For a discussion of their results see section 3.1 in RMFL.

Table 1: MAIN PANEL COINTEGRATION REGRESSION (1980-2004). DEPENDENT VARIABLE: LOG REER

	Ricci et al. (2013)	Demographic sample			
	(1)	(2)	(3)	(4)	(5)
	DOLS	DOLS	CCEP	DOLS	CCEP
Log old dependency ratio				0.290	0.276
				(0.014)	(0.000)
Log commodity terms of trade	0.548	0.656	0.442	0.654	0.401
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Net foreign assets to trade	0.0442	0.0423	0.0361	0.0408	0.0249
	(0.003)	(0.005)	(0.000)	(0.006)	(0.006)
Gov consumption to GDP	2.937	3.110	2.580	2.696	2.206
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Log productivity differential	0.185	0.217	0.240	0.175	0.203
	(0.004)	(0.001)	(0.000)	(0.005)	(0.000)
Trade restriction index	0.139	0.138	0.198	0.0975	0.157
	(0.009)	(0.017)	(0.000)	(0.103)	(0.000)
Price controls	-0.0448	-0.0453	-0.0396	-0.0380	-0.0321
	(0.045)	(0.042)	(0.014)	(0.109)	(0.058)
Observations	861	807	942	807	942
Number of countries	48	45	45	45	45
CD			22.37		8.635

p-values in parentheses

Note. All specifications include two dummies to account for periods of capital account liberalization for Indonesia (value 1 for periods 1980-82, 0 otherwise) and for Argentina (value 1 for periods 1991-2001, 0 otherwise). Column (1) replicates Table 4.(2) of Ricci et al. (2013) using the full "large sample". Columns (2) to (5) use the "demographic sample" (i.e."large sample" without Taiwan, Pakistan, Morocco). The underlying model of the columns with DOLS methodology ((1), (2) and (4)) is equation (2.1) as in Ricci et al. (2013): country fixed effects, 1 lead and 1 lag ($p = 1$), Newey-West standard errors to correct for serial correlation with 1 lag order of autocorrelation. The remaining columns ((3) and (5)) report results obtained applying CCEP methodology where the model is equation (2.3) with pooled constant: since in this specification there are neither leads nor lags, nor variables in first difference, the number of observations is greater (942 vs 807). Columns (4) and (5) are the same of columns (2) and (3) respectively except that they include the log of the old-dependency ratio (relative to trading partners) as a regressor. The last row of the table presents Pesaran (2015)'s CD test of the null hypothesis that the error term in (2.3) is weakly cross-sectional dependent: $\mathbb{E}\left[\zeta_{i,t}\zeta_{j,t}\right] = 0, \forall t, i \neq j$. Under the null, the CD test statistic is distributed N(0,1). In both cases the null of cross-sectional dependence is strongly rejected.

on the log of old dependency ratio obtained with DOLS (column (4)) while the p-value is now even closer to zero.[14]

[14]Using the CCEP methodology, Groneck and Kaufmann (2017) find results of a similar order of magnitude. In a panel of 15 OECD economies between 1970 and 2009, they find that a 1% increase

3 Age-specific sectoral consumption data

The age-dependent sectoral consumption shares represent both motivating evidence for a demand channel affecting the relative prices as well as the key parameters in our theoretical model. Using country-based survey data, we have built a more systematic evidence on the age-specific sectoral consumption than the one in the literature.[15]

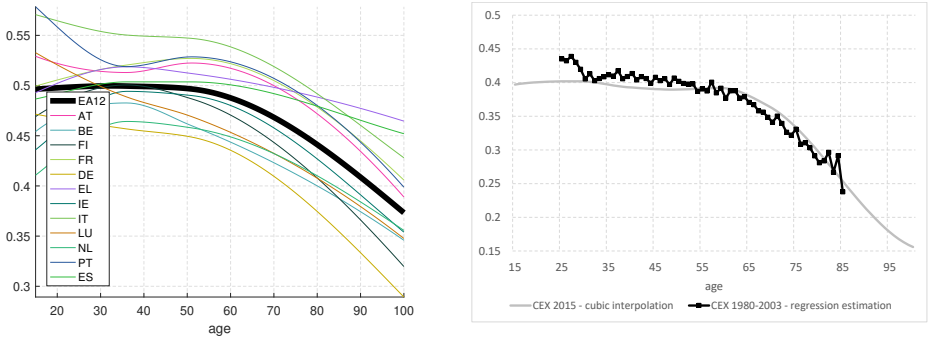
Figure 1 shows the age-dependent shares of consumption expenditure devoted to tradable goods and services based on national households' surveys for the twelve core Euro-area countries and United States. As only eleven consumption macro-categories were available for European countries, we first kept the same level of detail in the categorization for US as in the European data. This is captured by the continuous lines resulting from our cubic interpolation on the available data-points for the last available years when we compiled the dataset (2010 for Europe, 2015 for US). Secondly, we analyzed the different cross-sections of US households for all years between 1980 and 2003 provided in the data-set compiled by Aguiar and Hurst (2013) based on the Consumer Expenditure Survey (CEX), using a more granular classification of consumption categories into tradable and nontradable.[16] As detailed in Appendix B.1 we run a regression of the share of consumption on tradables, on a constant, age and year dummies. The marked-black line in panel (b) reports the estimated coefficient values on the constant and the age-dummies.

This chart reveals that the age-pattern is very similar for both Europe and US: households devote less of their consumption expenditure to tradable goods and ser-

of the old dependency ratio inflates the relative price of nontradables by 0.34%, thus explaining about one fifth of the average increase in relative prices between 1990 and 2009.

[15] Groneck and Kaufmann (2017) estimate that the share of aggregate nontradable consumption of the older people (aged 65+) is on average about eight percentage points higher than in the case of the younger people (aged 15-64) for US in 2011. Braude (2000) finds a difference of 20 percentage points for US in 1990 among the two shares. Ewijk and Volkerink (2012) report evidence for Netherlands with a corresponding difference amounting to 13 percentage points. Hobijn and Lagakos (2003), Börsh-Supan (2003), Ewijk and Volkerink (2012) provide more detailed evidence for US, Germany and Netherlands respectively. Lürmann (2005) estimates the composition of consumption age-profiles using panel data for Germany.

[16] The note under Figure 1 specifies how the consumption categories are classified to be either tradable or nontradable in the baseline. Moreover, for the US (panel (b), marked-black line) we used a more detailed classification based on 49 categories (as from Figure 15).



(a) Europe (EA12)

(b) United States (US)

Figure 1: Age dependent tradable shares of (private) consumption expenditure, α_j

Note. **Panel (a)**. *EU data source*: EUROSTAT, 2010 "Structure of consumption expenditure by age of the reference person (COICOP level 2) (1 000) [hbs_str_t225]" in 2010, i.e. the average private consumption expenditure (measured in euro/PPS). Tradables: food, clothing, furniture and equipment, transports, communications. Nontradables: housing, health, culture and entertainment, education, restaurants and accommodation. Age classes on which cubic interpolation is performed: 0-29, 30-44, 45-59, 60+. The EA12 profile (continuous thick line) is a GDP weighted average of the single European countries' profiles (GDP weights computed using EUROSTAT data). **Panel (b)**. *US data source*: Consumer Expenditure Survey (CEX). The *continuous grey* line is our interpolation on CEX, 2015, "Table 1300. Age of reference person: Shares of annual aggregate expenditures and sources of income". Tradables: food at home, alcoholic beverages, furnishings and equipment, apparel and services, transportation, tobacco products and smoking supplies. Nontradables: food away from home, housing minus furnishings and equipments, healthcare, entertainment, personal care products and services, reading, education. Age classes on which cubic interpolation is performed:: 25-, 25-34, 35-44, 45-54, 55-64, 65-74, 75+. The *marked black* line reports the estimated coefficient values on the constant and the age dummies of an OLS regression of the share of consumption on tradables on a constant, age dummies and (normalized) year dummies when we employ the dataset complied by Aguiar and Hurst (2013). Further details and analysis in Appendix B.1.

vices as they grow older. While the share is fairly constant for the young ages, the average discrepancy between those aged 50 and those aged 85 or more is about 10 percentage points according to European data and more than 15 percentage points according to US data. This feature is robust to different years of analysis.[17]

As detailed in Appendix B.1, at older ages households decrease the share of consumption on most categories, especially on "transportation", in favor of "health-

[17]Our results for European countries in the other years in which EUROSTAT data are available (1988, 1994, 1999, 2005), not reported here, confirm the age-pattern of Figure 1, panel (a).

care" and "housing (excluding furnishing and equipment)".[18]

4 Model

We consider a two-country OLG model that expands on the work by Domeij and Flodén (2006) and Krueger and Ludwig (2007) to incorportate two sectors. Each country is populated by overlapping generations of households that solve a life-cycle consumption problem. Only two goods are produced and consumed: a homogeneous good that can be traded between countries (T) and serves as numeraire, and a country-specific good that is not traded between countries (N). The production technology is identical across countries. Capital is perfectly mobile across countries and sectors, and can only be produced by means of tradable goods. Labor is immobile across countries and imperfectly mobile across sectors. The model is real and abstracts from nominal frictions. Demographics as well as age specific consumption shares are exogenous. In the model, one period corresponds to one year. The upcoming sections of the model are described for the domestic economy and hold with symmetry for the foreign economy.

Households. Households start their economic life at age 15, retire at age 65 and live at most until age 100. We denote the 15th year of life by $j = 0$, the retirement age by $j_r = 50$ and the highest possible age of life as $J = 85$. Households face an idiosyncratic, time-dependent (conditional) probability to survive from age $j-1$ to age j denoted by $s_{t,j}$ such that for each age $j = 0, 1, \cdots J$, the demographic size in period t, $N_{t,j}$, is given recursively by:[19]

$$N_{t,j} = N_{t-1,j-1} s_{t,j} \tag{4.1}$$

[18] See "2015 Aggregate Expenditure Shares Tables, Age of reference person" available at https://www.bls.gov/cex/csxashar.htm where the reader is redirected for the complete list of subcategories. Housing and Transportation contain sub-categories whose classification into either tradadable or nontradable might be doubtful. We control for this using the disaggregated classification of Figure 15.

[19] Following Domeij and Flodén (2006), the survival probabilities $s_{t,j}$ are retrieved using data on $N_{t,j}$ for all t, j. Therefore, due to migration, the survival probabilities can exceed 1, see section 5.

For each period t the life-cycle problem is such that the representative household born in t chooses consumption in each sector $c^N_{t,j}$, $c^T_{t,j}$ and the amount of assets to hold the sequent period $a_{t+j+1,j+1}$ for each age $j \in \{0, 1, 2, \cdots J\}$ under the assumption of perfect domestic annuities market;[20] how to allocate in each sector an exogenously given amount of hours to work, $h_{t,j}$ choosing $h^N_{t,j}$, $h^T_{t,j}$; how much to give as bequest $b_{j_b} \geq 0$ (to new entrants in the labor market, i.e. to those aged $j = 0$) when the bequest age $j = j_b$ comes given the importance of bequest as captured by the parameter Γ, solving the following problem:

$$\max_{c^N_{t+j,j}, c^T_{t+j,j}, h^N_{t+j,j}, h^T_{t+j,j}, a_{t+j+1,j+1}, b_{t+j_b,j_b}} \left\{ \sum_{j=0}^{J} \beta^j \pi_{t+j,j} \log c_{t+j,j} + \Gamma \beta^{j_b} \pi_{t+j_b,j_b} \log b_{t+j_b,j_b} \right\}$$

subject to

$$\begin{aligned}
c_{t+j,j} &= (c^T_{t+j,j})^{\alpha_j} (c^N_{t+j,j})^{1-\alpha_j} && (4.2)\\
h_{t+j,j} &= \left[\theta^{-\frac{1}{\epsilon}} (h^T_{t+j,j})^{\frac{\epsilon+1}{\epsilon}} + (1-\theta)^{-\frac{1}{\epsilon}} (h^N_{t+j,j})^{\frac{\epsilon+1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon+1}} && (4.3)\\
a_{t+j+1,j+1} &= \frac{a_{t+j,j}(1+r_{t+j})}{s_{t+j,j}} - c^T_{t+j,j} - P^N_{t+j} c^N_{t+j,j} - b_{t+j_b,j_b} I(j = j_b) + y_{t+j,j}\\
y_{t+j,j} &= (1-\tau_{t+j})(w^N_{t+j} h^N_{t+j,j} + w^T_{t+j} h^T_{t+j,j}) I(j < j_r) + d_{t+j,j} I(j \geq j_r)\\
a_{t+J+1,J+1} &= 0\\
a_{t,0} &\quad \text{given}
\end{aligned}$$

[20] The assumption of "perfect annuities market" means that the agents within each age group j agree to share the assets of the dying members of their age group among the surviving members. Using the notation just introduced, consider those that at time t are aged j. The total amount of assets of the dying members is: $a_{t,j}(1 - s_{t,j})N_{t-1,j-1}$, while the number of surviving members is: $N_{t,j} = N_{t-1,j-1} s_{t,j}$. Hence, in the budget constraint the asset holding in period $t+1$ will depend on what has been accumulated plus this sort of 'equal gift' from the dying members given the real interest rate (r_t) at which these assets can be invested (minus consumption plus income):

$$\begin{aligned}
a_{t+1,j+1} &= a_{t,j}(1+r_t) + \frac{a_{t,j}(1+r_t)(1-s_{t,j})N_{t-1,j-1}}{N_{t-1,j-1} s_{t,j}} - c_{t,j} + y_{t,j}\\
&= \frac{a_{t,j}(1+r_t)}{s_{t,j}} - c_{t,j} + y_{t,j}
\end{aligned}$$

which is the budget constraint written in the main text.

where $\pi_{t+j,j} = \prod_{k=0}^{j} s_{t+k,k}$ represents the unconditional survival probability with $s_{t,0} = 1$; β is the discount factor; $I(\cdot)$ is an indicator function; $d_{t+j,j}$ denotes the pension benefit.

Prices w_t^T, w_t^N, r_t, P_t^N denote the real wage in the tradable and non-tradable sector, the real interest rate on assets, and the the relative price of nontradables respectively and are taken as given by the household. The household's labor supply in efficiency units, $h_{t+j,j} = h_j$ is exogenous and depends on age but is constant over time. Particularly, it varies because of changes in productivity and labor market participation similarly to what is assumed in Domeij and Flodén (2006).

The two distinctive features of our model are represented by constraints (4.2) and (4.3): the parameter $0 < \alpha_j < 1$ denotes the age-dependent share of consumption expenditure devoted to tradables as captured by our empirical findings (see Figure 1); with $0 < \theta < 1$, the parameter ϵ denotes the degree of substitutability between hours supplied in the two sectors (both at the individual and at the aggregate level): $\epsilon \to \infty$ entails perfect labor mobility; $\epsilon \to 0$ is the case of no mobility (see section 7 for an analysis of this assumption).[21]

Firms. The representative firm in each sector $s \in \{T, N\}$ and in each period t hires (hours in efficiency units of) labor L_t^s at a given hourly wage rate w_t^s and rents

[21] The modeling choice of imperfect mobility between sectors allows demand factors (such as the change in demand composition induced by aging) to influence the structure of long-run relative prices. This is consistent with the empirical finding that wages tend not to be equalized between sectors in the long-run (see Cardi and Restout (2015)). In neoclassical models with perfect factor mobility the long-run relative price of nontradables is independent of consumer demand patterns (see Obstfeld and Rogoff (1996), ch. 4). There are other ways to allow for demand factors to matter (e.g. having diminishing returns to scale in at least one sector, see Galstyan and Lane (2009); or assuming that an economy is partially shut off from world capital markets, see Froot and Rogoff (1994); different capital elasticity between sectors can also work in the context of an OLG model, see section 4.1). We chose this due to its intuitiveness and close link with recent literature.
In particular, Cardi and Restout (2015) provide evidence that sectoral wages do not equalize in the long-run. Assuming a CES aggregator for sectoral hours as in (4.3) means that households have a preference to diversify labor despite wage differences between sectors. This can be thought as some degree of preference for the *status quo* when switching sectors (see Dix-Carneiro (2014)). More broadly, it can be thought to capture structural forces in an economy, including compositional differences of the work-force between sectors, that might be responsible for the long-run persistence of sectoral wage differences detected in the data. Other works employing a CES aggregator to capture imperfect sectoral labor mobility include: Kim and Kim (2006), Bouakez et al. (2009), Iacoviello and Neri (2010), Bouakez et al. (2011), Altissimo et al. (2011), Groneck and Kaufmann (2017), Cantelmo and Melina (2018).

capital K_t^s at real interest rate r_t subject to yearly depreciation rate δ to solve:

$$\max_{K_t^s, L_t^s} \left\{ P_t^s (K_t^s)^{\psi^s} (Z_t^s L_t^s)^{1-\psi^s} - w_t^s L_t^s - (r_t + \delta) K_t^s \right\}$$

where P_t^T is normalized to one and $0 < \psi^s < 1$ determines the capital intensity.

Government. The government runs a pay-as-you-go, defined-benefits pension system. There is a flat pension tax on the average labor income, and the tax revenue is distributed as lump-sum pension benefits among all retirees aged 65 and above. Thus, the government sets a tax rate τ_t such that its budget balances in each period:

$$\tau_t = \frac{d_t \sum_{j=j_r}^{J} N_{t,j}}{w_t^T L_t^T + w_t^N L_t^N}$$

Clearing. The labor market in each sector $s \in \{T, N\}$ and the market for nontradables clear in each period t:

$$\begin{aligned} L_t^s &= \sum_{j=0}^{J} h_{t,j}^s N_{t,j} \\ (K_t^N)^{\psi^N} (Z_t^N L_t^N)^{1-\psi^N} &= \sum_{j=0}^{J} N_{t,j} c_{t,j}^N \end{aligned}$$

The initial asset assigned to the representative household entering the labor market at the beginning of period t ($a_{t,0}$) is determined by the amount of bequest left by the end of period $t-1$. The total amount of bequest is split equally among all the households entering the labor market such that:

$$a_{t,0} = \frac{b_{t,jb} N_{t-1,jb}}{N_{t,0}}$$

The international capital market clears where '$*$' denotes the rest of the world):[22]

$$K_t^T + K_t^N + (K_t^T)^* + (K_t^N)^* = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j} + \sum_{j=0}^{J} a^*_{t+1,j+1} N^*_{t,j} + a_{t,0} N_{t,0} + a^*_{t,0} N^*_{t,0}$$

The RER. The RER between two countries is the relative purchase price of the common reference basket of goods which can be compared by means of a common numeraire. It can be decomposed in the two components that pertain to the two sectors of the economy (time subscript t is omitted for brevity):

$$RER \equiv NER \frac{P}{P^*} = \underbrace{\left(NER \frac{P^T}{P^{T*}} \right)}_{RER^T} \underbrace{\left(\frac{P/P^T}{P^*/P^{T*}} \right)}_{RER^N}$$

where NER is the (bilateral) nominal exchange rate defined as foreign currency per unit of domestic currency; P and P^* are the price indexes in the respective country. We have decomposed RER in two parts: the former pertains to the tradable side of the economy (RER^T); the latter pertains to the nontradable one (RER^N). By this definition, *an increase in* RER means a *real appreciation* for the domestic country.

In our setting, money does not play a role: there are no nominal rigidities and no feedback from the monetary to the real side of the economy. Therefore, without loss of generality, we fix NER to 1. Furthermore, by clearing of the tradable goods market, the law of one price holds for tradables. We normalize the tradable good's price to unity so that $P^T = P^{T*} = 1$. Therefore, the tradable component of RER is always equal to one in equilibrium, so that variations of RER are explained only by variations in its nontradable component (RER^N). Admittedly, our model is suitable to capture *long-run* variations of the RER.

More precisely, RER^N is the ratio of a function of the relative price of non-tradable to tradable goods in the domestic country to that in the foreign country.

[22] The market for tradables clears too (by Walras' law this is a superfluous condition in terms of computation once all the others above are satisfied):

$$C_t^T + C_t^{T*} + K_{t+1} = (1-\delta) K_t + Y_t^T + Y_t^{T*}$$

where: $Y_t^T = (K_t^T)^{\psi^T} (Z_t^T L_t^T)^{1-\psi^T}$, $Y_t^{T*} = (K_t^{T*})^{\psi^{T*}} (Z_t^{T*} L_t^{T*})^{1-\psi^{T*}}$, $C_t^T = \sum_{j=0}^{J} c_{t,j}^T N_{t,j}$, $C_t^{T*} = \sum_{j=0}^{J} c_{t,j}^{T*} N_{t,j}^*$, $K_t = K_t^T + K_t^N + K_t^{T*} + K_t^{N*}$.

Under our assumptions:

$$RER = RER^N = \frac{\frac{P(P^T,P^N)}{P^T}}{\frac{P^*(P^{T*},P^{N*})}{P^{T*}}} = \frac{P(1,P^N)}{P^*(1,P^{N*})}$$

Hence, the aggregate price indexes in the two countries (P, P^*) will be increasing functions of the respective relative prices of nontradables (P^N, P^{N*}).

In our framework in which agents are heterogeneous with respect to age, the price index associated to each age-dependent consumption bundle can be solved through the utility maximization problem. Following a standard definition (see Obstfeld and Rogoff (1996) section 4.4.1.1), given the household utility function above: $\log c_j$, we determine the age-dependent consumption-based price index p_j for each age $j = 0, 1, \cdots, J$ as the minimum expenditure $c_j^T + P^N c_j^N$ such that the consumption bundle $c_j = (c_j^T)^{\alpha_j}(c_j^N)^{1-\alpha_j} = 1$, given P^N. It can be shown that the following holds:

$$p_j = \frac{\left(P^N\right)^{1-\alpha_j}}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}}$$

However, when it comes to the aggregate price index a higher level of arbitrariness is allowed by the fact that our model does not feature a single representative agent. In principle, any aggregate price index computed on the basis of P^N and p_j could be justified as the model's counterpart to real-world price index data. An obvious candidate is the aggregate price index composed as a weighted (arithmetic) average of each age-dependent price index with weights given by the share of people in each age-bin in each period:

$$\text{Index 1}: \qquad P = \sum_j p_j \omega_j \tag{4.4}$$

where the age-dependent weights are given by the shares of people in each age-group j: $\omega_j = N_j / \sum_j N_j$.

To be closer to the existing (theoretical and empirical) literature, which is generally embedded within the representative agent framework with no age-dependent

consumption shares, we will also provide results when the aggregate price index is simply measured as the relative price of nontradables corrected for the aggregate share of nontradables in the economy, see e.g. Berka et al. (2018):

$$\text{Index 2}: \qquad P = (P^N)^{1-\gamma}, \quad \gamma = \sum_j \alpha_j \omega_j \tag{4.5}$$

4.1 Inspecting the mechanism: a two-period version of the model

To highlight the mechanism through which demographics affects the RER, we consider a two-periods version of the model. The only difference as compared to the general framework described above is that the household is alive for two periods: age 0 as a worker, age 1 as retiree. Given the real interest rate r, the stationary equilibrium in each country is fully determined by the relative wage in the labor market and characterized by the following relative supply and demand:

$$L^S \quad : \qquad \frac{w^N}{w^T} = \left[\frac{\theta}{1-\theta}\frac{h^N}{h^T}\right]^{\frac{1}{\epsilon}} \tag{4.6}$$

$$L^D \quad : \qquad \frac{w^N}{w^T} = \Lambda \left(\frac{h^N}{h^T}\right)^{-1} \tag{4.7}$$

where

$$\Lambda = \left\{ \frac{1 + s_1\beta\left[1 + \Gamma(1-(1+r)^2)\right]}{\left[1-\alpha_0 + (1-\alpha_1)s_1\beta(1+r)\right](1-\psi^N)\left(1-\tau\frac{r}{1+r}\right)} - 1 \right\}^{-1} \tag{4.8}$$

The labor supply intuitively depends on the hours elasticity of subsitution ϵ. The demand slope depends on demographics in two ways: via the the conditional survival probability of being alive from period 0 to 1, $s_1 = n_1/n_0$, where we denote the number of workers and retirees in stationary equilibrium respectively (see equation (4.1)) by n_0 and n_1 (i.e. s_1 is the old dependency ratio); and via the tradable demand shares in young and old age α_0 and α_1.

As demand meets supply, the resulting equilibrium relative wage is:

$$\frac{w^N}{w^T} = \left(\frac{\Lambda\theta}{1-\theta}\right)^{\frac{1}{1+\epsilon}} \tag{4.9}$$

which in turn allows us to identify the equilibrium relative price using firms' optimality conditions (see Appendix A.3 for details):

$$P^N = \left[\frac{Z^T}{Z^N}\frac{w^N}{w^T}\frac{(1-\psi^T)\left(\frac{\psi^T}{r+\delta}\right)^{\frac{\psi^T}{1-\psi^T}}}{(1-\psi^N)\left(\frac{\psi^N}{r+\delta}\right)^{\frac{\psi^N}{1-\psi^N}}}\right]^{1-\psi^N} \tag{4.10}$$

From this expression we see that one country's relative price of nontradable goods depends, even in the absence of sectoral differences in capital intensity ($\psi^T = \psi^N$) or in labor productivity ($Z^T = Z^N$), on the relative wage. Through the relative wage, the relative price of nontradables depends on aging (higher s_1), age specific consumption patterns ($\alpha_1 \neq \alpha_0$) and on the real interest rate r, which is in turn affected by demographics. The two-period model shows that the way aging, an increase of the old-dependency ratio s_1, can impact the equilibrium relative price is by shifting the relative demand of labor towards the production of nontradables (i.e. by changing Λ, see (4.7), (4.8)) determining a change in the equilibrium relative wage (see (4.9), (4.10)). Moreover, a relative demand shift can affect the equilibrium relative wage only if the relative supply of labor is non-flat, i.e. labor is imperfectly mobile between sectors, $\epsilon < \infty$ (see (4.6)). It is worth noticing that the equilibrium relative price also depends on the relative labor productivity Z^T/Z^N (both scaled by the labor share in the Cobb-Douglas production function, $1-\psi^N$) which captures the standard Balassa-Samuelson effect (see (4.10)). Exogenous changes in the relative labor productivity do not affect the relative demand for labor.[23]

Figure 2 illustrates the effect of aging between the initial (1960) and the final (2050) steady state on relative wages in EU and US (the calibration is such

[23] Formally, this is due to the assumption that consumption in the two sectors is aggregated via a Cobb-Douglas function, see the household's problem in section 4. That is the sectoral consumption share does not vary with the relative price as the elasticity of substitution in consumption between the goods of the two sectors is equal to 1.

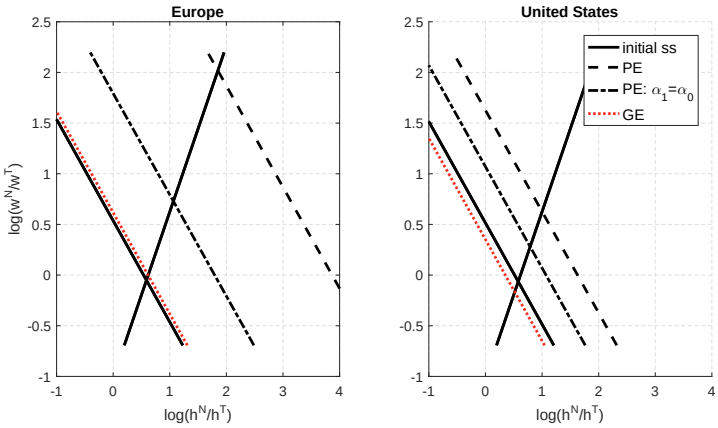


Figure 2: Labor market equilibrium in the two countries (EU *vs* US)

Note. Two-period model calibrated with the old-dependency ratio of EU and US in 1960 (initial steady state, continuous lines) and in 2050 (final steady state, other lines) from Figure 16. The final steady state is analyzed under different conditions: partial equilibrium (PE), i.e. fixing the real interest rate at the initial steady state value (dashed lines) and assuming age-invariant consumption shares (dashed-dotted lines); general equilibrium (GE), i.e. letting the real interest rate endogenously adjust.

that the two countries differ only in terms of aging, see Appendix A.3 for calibration details). Going from 1960 to 2050 aging is more pronounced for EU: the old-dependency ratio becomes about 3.45 times higher in EU, and only 2.28 times higher in the US. The aging process with the ensuing increase of the aggregate saving rate leads the (annualized) real interest rate to go from 5.45% in 1960 to 3.74% in 2050. The continuous lines show the initial steady state equilibrium. We consider three scenarios: *partial equilibrium* in which aging takes place and the interest rate is kept at the initial steady state value; *partial equilibrium* in which differences in demand composition across age groups do not play a role ($\alpha_1 = \alpha_0$); and a third *general equilibrium* scenario with age-specific consumption shares.

The slope of the labor demand increases with aging. That is, Λ is an increasing function of the old dependency ratio.[24] The study of this slope reveals two channels in partial equilibrium, i.e. for given real interest rate.

First, the *age-dependent consumption composition* channel: when retirees have

[24]This holds under the condition that $(1-\alpha_0)/(1-\alpha_1) < 1+r$ which is always satisfied for all real interest rate values $r > 0$ both with $\alpha_0 > \alpha_1$ (which is justified empirically - see Figure 1) and with $\alpha_1 = \alpha_0$

a higher share of nontradables in their consumption basket in comparison to workers ($\alpha_1 < \alpha_0$), an increase in the relative number of retirees (s_1) implies a shift of consumption demand in favor of nontradable goods which makes firms demanding relative more labor in the nontradable sector.

Second, even in the absence of consumption composition differentials across ages, aging leads to an increase in the relative price of nontradables due to a *savings* channel. An increase in the survival probability (s_1) makes workers save more in expectation of a longer retirement period.[25] For given positive prices and real interest rate ($r > 0$) a stationary equilibrium with higher saving rate is characterized by higher aggregate consumption.[26] A higher aggregate consumption needs to be met by more work. Higher demand for nontradable needs to be faced nationally, while higher demand of tradables can be responded to also by imports, hence the nontradable wages increase.[27]

Aging holds across economies. Hence the above two channels will come into effect with the same sign for all countries, even though the extent and timing might differ. Importantly, it also means that there are general equilibrium effects that point in the same direction. In particular, as aging induces a higher aggregate saving rate, a stationary equilibrium with a higher fraction of elderly will have a lower real in-

[25] Considering the case of no bequest and no pension: $\Gamma = \tau = 0$, the budget constraint of the representative household is simply: $a_1 + c_0^T + P^N c_0^N = y_0$. Then, the aggregate saving rate ζ (equals the individual rate in this setting) is: $\zeta = (n_0 a_1)/(n_0 y_0) = a_1/y_0 = 1 - (c_0^T + P^N c_0^N)/y_0 = 1 - (c_0^T/\alpha_0)/y_0 = \beta s_1/(1+\beta s_1)$ where we have used the first order conditions (A.10), (A.11) and (A.12) with $j = 0, 1$. In a stationary equilibrium aging, increase in s_1, unambiguously increases the saving rate.

[26] A unit of consumption forgone today returns more than a unit of consumption tomorrow as long as $r > 0$. In a stationary equilibrium the unit of consumption the young gives up today shows up as a greater than one increase of consumption for the old today.

[27] When the government is allowed to have a role ($\tau > 0$) there is also a third channel: *pension system* channel. A PAYGO pension system with fixed replacement rate $\bar{d}$ requires that the tax rate τ increases with aging (see (A.8)). As pension transfers crowd out savings, the increase in the tax rate implied by aging will tend to shift the relative demand of labor inwards in the nontradable sector (Λ is decreasing in τ) for the same reasoning of the savings channel.

terest rate as determined in the international market for perfectly mobile capital.[28][29]

As a result, the quantitative impact of the demand channels above will be diluted. A lower real interest rate makes savings less profitable so that a given unit of savings of the young gives a lower level of consumption for the old thus reducing the aggregate consumption of nontradables with associated less relative demand for labor in their sector. This holds independently of whether consumption shares are age-dependent. If the elderly also have a higher share of consumption devoted to nontradables as compared to the young ($\alpha_1 < \alpha_0$), the relative demand for labor in the nontradable sector is further dampened with the decrease of the real interest rate.[30]

5 Calibration

In the full quantiative model we once again consider US and EU,[31] evaluated at an initial stationary equilibrium in 1960 and a final stationary equilibrium in 2050. To isolate the impact of demographic change, we assume that the two areas only differ in terms of demographics.

[28] Consider a closed economy with $\Gamma = \tau = 0$ and equal capital intensity between sectors $\psi^N = \psi^T = \psi$. Aggregate profits are zero: $Y = (r+\delta)K + wL$. Aggregate capital equals aggregate savings: $K = n_0 a_1$, while the aggregate labor costs equal the aggregate labor income: $wL = n_0 y_0$. Therefore, the output-capital ratio is $Y/K = wL/K + r + \delta = 1/\zeta + r + \delta$. Using the firms' first order condition with Cobb-Douglas production function: $r + \delta = \psi(Y/K)$, the equilibrium real interest rate is: $r = \frac{\psi}{(1-\psi)\zeta} - \delta$ where the saving rate, derived in footnote 25, is $\zeta = \beta s_1/(1+\beta s_1)$. That is, in a stationary equilibrium aging (increase in s_1) unambiguously reduces the real interest rate by increasing the saving rate.

[29] See Papetti (2020) for explanation and quantification of the various channels determining a downward impact of aging on the real interest rate in a large-scale OLG model of the type presented in section 4.

[30] Notice, again for the case $\Gamma = \tau = 0$, that Λ is an increasing function of, not only the old-dependency ratio s_1, but also the real interest rate r and that a given decrease of r makes Λ decrease more the higher the fraction of consumption devoted to nontradables by the old $(1-\alpha_1)$. Furthermore, consider the relative aggregate consumption of nontradables: $P^N C^N/C^T = P^N(n_0 c_0^N + n_1 C_1^N)/(n_0 c_0^T + n_1 c_1^T) = (1-\alpha_0+(1-\alpha_1)\beta(1+r)s_1)/(\alpha_0+\alpha_1\beta(1+r))$. It can be shown that this expression is an increasing function of both s_1 and r if and only if $\alpha_0 > \alpha_1$. If $\alpha_1 = \alpha_0 = \alpha$, then the relative aggregate consumption would be constant at $(1-\alpha)/\alpha$.

[31] We use the twelve core euro-area countries (EA12): Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Spain (ES). EU parameters and variables are obtained as year specific GDP weighted averages of the single countries' parameters or variables.

For each year, the number of people in each age-group j, N_j, is taken from UN (2017). Based on N_j we apply (4.1) to retrieve the survival probabilities in each stationary equilibrium. As shown in Appendix figure 17, the population sizes of the two economies considered are very close which ensures that general equilibrium effects induced by demographic change are not dominated by the prominence of one economy. The right panel of the same figure shows the resulting unconditional survival probabilities ($\pi_j = \prod_{k=0}^{j} s_k$ with $s_0 = 1$) which shifts outwards from 1960 to 2050 as the probability of remaining alive for each age-group increases dramatically. Survival probabilities can exceed 1 due to migration. We assume that immigrants arrive without assets and are adopted by domestic households.[32]

Individual labor supply in efficiency units, h_j, is interpolated using data from Domeij and Flodén (2006),[33] accounting for the age profile of both productivity and labor market participation. It is assumed that individuals enter the world as workers at age 15 and retire at age 65 as standard in the literature.[34] The net replacement rate $\overline{d}$ is set to 45% of the average net workers earnings throughout the life-cycle, about what is reported by OECD data for US in 2014.

Changes in the demographic distribution affect the aggregate demand composition via the interaction of the number of people in each age-group, N_j, with the age-varying shares of consumption devoted to tradable goods, α_j (calculated as described in Figure 1's footnote). For both EU and US, after about age 60, less and less consumption expenditure is devoted to tradable items. There seems to be a level discrepancy between EU and US, the latter consuming systematically more nontradable goods for every age which is likely due to the use of data for *private* consumption expenditures: the public welfare system, which includes mostly nontradables, is notably less comprehensive in the US as compared to EU. As US consumption shares better capture actual overall consumption, we assume that both areas have the age-varying consumption shares of the US in the baseline calibration.

The output elasticity to capital in the Cobb-Douglas production function is assumed to be the same in both sectors and countries, $\psi^T = \psi^N = \psi^{T*} = \psi^{N*}$, set to

[32] See Domeij and Flodén (2006) for a short motivation of this assumption.
[33] See Figure 19 in Appendix
[34] See e.g. Kara and von Thadden (2016) for euro-area, Krueger and Ludwig (2007) for US in line with the values in Carvalho et al. (2016)).

Table 2: BASELINE CALIBRATION: INITIAL STEADY STATE

Parameter	EU	US	Note
$\psi^T = \psi^N$	0.34		output elasticity to capital (Cobb-Douglas, both sectors)
δ	0.0778		capital depreciation rate (target: $K/Y = 2.57$, $I/Y = 0.2$)
$\bar{d}$	0.45		net replacement rate: % of average net working earnings
Γ	0		preference for bequest
θ	0.35		tradable labor compensation, average 1970-2007 EUKLEMS
ϵ	0.61		degree of labor mobility, see Cardi and Restout (2015)
Z^T/Z^N	1.5		relative labor-augmenting technology, see Cardi and Restout (2015)
β	0.966		individual discount factor, set to target $r = \psi/(K/Y) - \delta$
J	86		terminal life-period (age 100)
j_r	50		retirement-age (i.e. age 65), see Carvalho et al. (2016) Tab. 2
α_j	Fig. 1 (US)		consumption shares in tradables. Source: CEX 2015
h_j	Fig. 19		labor supply in efficiency units. Source Domeij and Flodén (2006)
N_j	Fig. 17 (EU)	Fig. 17 (US)	1960 number of people in each age-group j. Source: UN (2017)

the standard value of about 1/3. From the firms' optimality conditions with respect to capital, this assumption implies that the aggregate capital-output ratio is the same in both countries.[35] We abstract from bequest (hence $\Gamma = 0$ which implies initial asset $a_0 = 0$) and target in the initial steady state the (weighted) average of capital- and investment-output ratios (K/Y, I/Y) as averages between 1970 and 2016. Using data from the World Development Indicators we target: $K/Y = 2.57, I/Y = 0.2$.[36] The depreciation rate δ is then obtained from the law-of-motion of capital ($K_{t+1} = (1-\delta)K_t + I_t$) in steady state: $\delta = 0.2/2.57 = 0.0778$. The real interest rate in the initial steady state is thus $r = \psi\frac{Y}{K} - \delta = 0.34/2.57 - 0.0778 = 0.0545$.

The degree of labor mobility between the two sectors $\epsilon = 0.61$ is obtained from the 'whole sample' estimate provided by Cardi and Restout (2015). We follow them also in calibrating the households' bias towards the tradables in the choice of sec-

[35] From (A.20) and (A.21): $\psi^T(Y^T/K^T) - \delta = r$ and $\psi^N(P^N Y^N)/K^N - \delta = r$. With $\psi^T = \psi^N$ it must be that the capital-output ratios in the two sectors $\left(K^T/Y^T, K^N/(P^N Y^N)\right)$ is the same and equal to the aggregate capital-output ratio. Furthermore, with $\psi^T = \psi^N = \psi^{T*} = \psi^{N*}$ and due to perfect mobility of capital the real interest rate is the same across countries,hence both countries must have the same aggregate capital-output ratio, by K/Y.

[36] Series used are: "Gross capital formation (constant LCU)", "Gross fixed capital formation (constant LCU)" and "GDP (constant LCU)" where we estimate the capital stock by applying the perpetual inventory method. The initial capital stock when 1970 is the base year is computed using the formula: $K_{1970} = I_{1970}/(g_I + \delta_K)$ where I_{1970} corresponds to the gross capital formation in 1970, g_I is the average growth rate between 1970 and 2016, while δ_K is set to 5% (see Cardi and Restout (2015)). The capital stock is then obtained via the neoclassical law-of-motion: $K_{t+1} = (1-\delta)K_t + I_t$. The base year is due to data availability 1970 for EU and 1960 for US. The weights for the final averages are the shares of GDP (constant LCU) in 2000.

toral labor supply, θ, matching the average tradable content of labor compensation between 1970 and 2007 according to EUKLEMS data.[37] We pick the mid-point between EU and US, about 0.35. Labor-augmenting technology is assumed to be 50% higher for firms in the tradable sector, in line with what Cardi and Restout (2015) calibrated for a representative OECD economy. Therefore we set $Z^T = Z^{T*} = 1.5$ and $Z^N = Z^{N*} = 1$.

Given all parameters in Table 2 except β we solve for the values of β and relative prices P^N, P^{N*} such that the capital market clearing condition at the world level and the nontradable goods clearing condition at the country level are satisfied in the initial steady state. This procedure gives an individual discount factor $\beta = 0.966$ while the resulting relative prices are $P^N = 1.0136$ for EU and $P^{N*} = 1$ for US (see first column of Table 3). The resulting life-cycle profiles are reported in Figure 3.

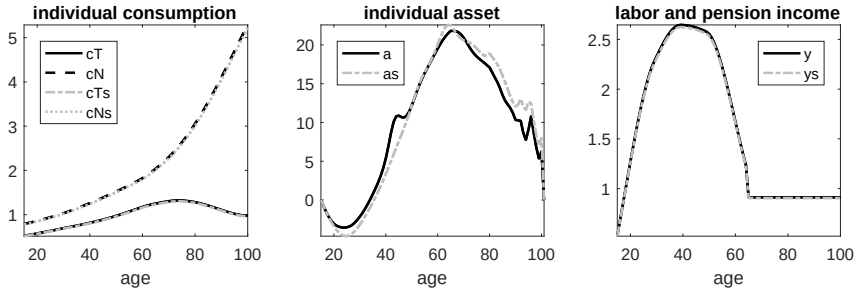


Figure 3: Initial steady state: life-cycle profiles: EU (black) *vs* US (grey)

Note. Calibration of Table 2.

Some observations deserve mentioning. First, individual profiles are similar in the two countries. The reason is that the demographic distribution of the two countries is very similar in 1960 (see Figures 16 and 17 in Appendix). Second, the individual consumption profiles (more so for nontradable goods) are increasing along the life-cycle. This is notably due to the assumption of perfect annuities market which ensures that the conditional survival probabilities do not appear in

[37] We use Cardi and Restout (2015) classification of EUKLEMS industries. *Tradable*: agriculture, hunting, forestry and fishing; mining and quarrying; total manufacturing; transport and storage and communication; financial intermediation. *Nontradable*: electricity, gas and water supply; construction, wholesale and retail trade; hotels and restaurant; real estate, renting and business activities; community social and personal services.

the inter-temporal condition thus tending to prevent a hump-shaped consumption profile.[38] Third, given that we abstract from bequest the asset holding profile starts at zero (the initial generation has no inheritance) and it is optimal to be indebted up until about age 35.

The final steady state is computed using the demographics of year 2050. We use the parameter values in Table 2 (except for the N_j which now correspond to year 2050) to solve for the new real interest rate and the new relative prices of nontradables in the two countries.

6 Results: quantifying the long-run effect of demographics on RER

We provide both the general and the partial equilibrium results. In the former case the real interest rate is endogenous satisfying the world market clearing condition. Whereas the real interest rate is fixed at the initial steady state level in the partial equilibrium case. For both partial and general equilibrium, three scenarios are considered: (1) the only exogenous change going from initial to final steady state is the demographic change ("demographics only"); (2) the only change is the exogenous constant compound growth of the relative labor-augmenting technology Z^T/Z^N ("relative productivity only"); (3) both demographic and relative productivity occur going from initial to final steady state ("demographics and relative productivity"). We report the six sets of results for the final steady state in Table 3 which also reports the comparative statics results for the two aggregate price indexes expressed in section 4 (see equations (4.4) and (4.5)) in the two countries and the RER obtained from both price indexes.

The breakdown in different result sets allows us to isolate our contribution with respect to previous work. In line with the literature, the biggest impact on each country's relative price of nontradables (P^N) is due to structural forces that lead to an increase in the relative productivity of the tradable sector – an effect generally

[38] These sectoral consumption profiles are consistent with our empirical estimates based on Aguiar and Hurst (2013)'s data-set and methodology. See Figure 14 in Appendix.

referred to as Balassa-Samuelson effect.[39] Since the growth rate of the relative labor productivity in the tradable sector is historically approximately constant across countries (see Figure 18 in Appendix), we fix the annual growth rate of Z^T/Z^N at 2% for both countries when we evaluate the case of relative productivity of tradable vs nontradable sector.

In addition to the relative productivity effect, we highlight the role of demographic change. Demographic change impacts the relative price of nontradables by how it impacts the relative wage (w^N/w^T) due to imperfect mobility of labor across sectors. In *partial equilibrium*, i.e. excluding general equilibrium effects on the real interest rate, demographic change only in the long-run leads to an increase in the relative price of nontradables in both countries by about 43.5% ($= 1.4549/1.0136 - 1$) in EU and 24.5% ($= 1.2448/1 - 1$) in the US. The effect of relative sectoral productivities coupled with demographic change leads to an approximately 4 times higher relative price of nontradables in the US and 4.6 times higher in EU. The bigger effect of demographics in EU is due to that EU ages relatively more: the number of retirees over workers becomes 3.45 times higher in EU; whereas it only becomes 2.28 times higher in the US.

This partial equilibrium effect of demographic change on the relative price of nontradables is dampened in *general equilibrium*. Aging leads to a decrease of the equilibrium world real interest rate of about 1.26 percentage points. At a lower real interest rate, the relative demand for nontradables tends to be lower, thus contributing to the lower level of the relative price of nontradables.[40] Once general equilibrium forces are taken into account, both the demographic change only and the demographic change plus relative productivity results are significantly damp-

[39] In our baseline calibration (in particular, under the assumption of equal capital intensity between sectors), firms' first order conditions imply: $P^N = \left(\frac{Z^T}{Z^N}\frac{w^N}{w^T}\right)^{1-\psi^N}$. Cardi and Restout (2015) show that a 1% increase in the relative price of nontradables can be decomposed into approximately 1.3% increase in the relative productivity of the tradable sector and a 0.3% decrease of the relative wage in the nontradable sector due to imperfect labor mobility between sectors. Another way to see this result is that the Balassa-Samuelson effect is imperfect. An increase of 1% in the relative productivity of the tradable sector is associated with an increase of only 0.77% ($\approx 1/1.3$) in the relative price of nontradables (not 1% as Balassa-Samuelson would predict), as the relative wage of the nontradable sector simultaneously decreases by about 0.23%.

[40] See the theoretical channels discussed in section 4.1.

ened.

Even if demographics matter little for the *level* of each country's relative price of nontrdadables it might matter significantly for the *relative levels* between countries, i.e. for the RER. While relative productivity is almost equal across advanced economies the demographic change differs. Indeed we observe that, even if the *ratio* of the relative price of nontradables does not change over time because the two countries have the same growth rate of the relative labor-augmenting technology, the ratio of the price indexes i.e. the RER still changes in the long-run because the *level* of the relative price of nontradables is different across countries in presence of differences in the consumption shares α_j and demographics N_j.

Table 3: INITIAL (1960) *vs* FINAL (2050) STEADY STATE: EFFECT OF DEMOGRAPHIC CHANGE (BASELINE CALIBRATION OF TABLE 2)

		Initial steady state	**Final steady state**		
			Demographics	**Relative productivity**	**Dem + Rel prod**
		General Equilibrium		***General Equilibrium***	
P^N	**EU**	1.0136	1.0496	3.2436	3.3590
	US	1.0000	0.9788	3.2002	3.1322
r	**world**	0.0545	0.0419	0.0545	0.0419
$RER = \frac{P^{EU}}{P^{US}}$	**Index 1 (eq. 4.4)**	1.0085	1.0382	1.0082	1.0501
	Index 2 (eq. 4.5)	1.0083	1.0454	1.0080	1.0572
Demographics	**Index 1 (eq. 4.4)**		2.95%		4.16%
impact on RER	**Index 2 (eq. 4.5)**		3.68%		4.88%
				Partial Equilibrium	
P^N	**EU**		1.4549	3.2436	4.6561
	US		1.2448	3.2002	3.9836
r	**world**		0.0545	0.0545	0.0545
$RER = \frac{P^{EU}}{P^{US}}$	**Index 1 (eq. 4.4)**		1.0992	1.0082	1.1123
	Index 2 (eq. 4.5)		1.1069	1.0080	1.1195
Demographics	**Index 1 (eq. 4.4)**		8.99%		10.33%
impact on RER	**Index 2 (eq. 4.5)**		9.78%		11.06%

The cross-sectional asymmetric effect on nontradables prices due to relative aging translates into a RER appreciation for the EU, that ages more. Depending on which index is used, the effect of demographic change within the *partial equilibrium* scenarios is about 9% to 10%. If one also considers equal-across-countries relative productivity (in terms of labor productivity growth) in partial equilibrium

the long-run effect of demographic change is slightly magnified by about 1.3 percentage points.[41] As a close consequence of the dynamics of the relative price of nontradables, the effect of demographic change on RER in *general equilibrium* is more than halved. The demographic only effect is about 3%. When also relative productivity forces are taken into account the demographic change effect on RER is again slightly magnified going to almost 5%.

7 Sensitivity analysis

We do sensitivity analysis on the two distinctive features of our model, namely the imperfect substitutability of working hours between sectors and the age-dependent sectoral consumption shares, in partial equilibrium (i.e. by fixing the real interest rate at its value in the initial steady state). We further explore the role of general equilibrium forces.[42]

The role of imperfect substitutability of labor between sectors. A distinctive feature of our model is the constraint on the allocation of hours between sectors (4.3) by which the working hours supplied by the households are not perfectly substitutable between the two sectors. Consider the condition implied by the optimal labor supply decisions (the time subscript is suppressed for brevity):

$$\frac{w^N}{w^T} = \left[\frac{\theta}{1-\theta}\frac{h_j^N}{h_j^T}\right]^{\frac{1}{\epsilon}} \tag{7.1}$$

The two polar cases are represented by $\epsilon \to 0$ and $\epsilon \to \infty$ (immobility and perfect labor mobility, respectively). When ϵ tends to zero, the working hours in each sector tend to be supplied in a fixed proportion of the exogenous h_j. On the contrary, when the relative supply of hours is infinitely elastic, sectoral hours are aggregated linearly ($h_j^T + h_h^N = h_j$) so that wages are equalized in equilibrium ($w^N/w^T = 1$). For intermediate cases, an increase of the relative labor supplied in the nontradable

[41] Notice that now the reference RER suited to identify the demographic change effect in the long-run is the one that would prevail if only relative productivity changes had occurred.

[42] For sensitivity to country-specific parameters see Appendix A.4

sector (h_j^N/h_j^T) is associated with an increase of the the relative wage (w^N/w^T).[43]

The smaller epsilon, the higher the increase in w^N/w^T corresponding to the same increase in h^N/h^T.

Figure 4 shows the effect of demographic change on the RER, for both Index 1 and Index 2 as defined in (4.4) and (4.5), and the ratio of relative prices in the two countries (P^N/P^{N*}) going from the initial to the final steady state when both steady states are computed under different values of ϵ (equal between countries) in partial equilibrium. The baseline case discussed in the previous section ($\epsilon = 0.61$) is represented by the square-marker.

It can be seen that lower labor mobility, lower ϵ, corresponds to a higher long-run demographic change effect on both RER indexes. Particularly, by setting ϵ close to zero we could generate a maximum long-run partial equilibrium demographic effect of about 15% appreciation for the EU. On the other hand, as ϵ increases, the effect of demographic change gradually fades away, as the relative labor supply flattens so that any shift of the relative labor demand in the nontradable sector corresponds to a smaller increase of the relative wage in the nontradable sector.[44]

Finally, Figure 4 shows that if both regions were as flexible as the US ('$*$' marker) in the labor market ($\epsilon = 1.8$, the country-specific estimates of Cardi and Restout (2015)), then the demographic change effect on RER in partial equilibrium would be much smaller, about half of what we have estimated in the previous section.

The role of age-dependent sectoral consumption shares. As an experiment, suppose we raise the tradable consumption shares at ages after retirement until they reach about the same level as at ages before retirement. For each scenario in Figure 5, panel (a) we compute both the initial and the final steady state, each time computing the demographic effect on RER in partial equilibrium ('demographics only' case) shown in Figure 5, panel (b). The baseline case marked by a square features about 9% RER appreciation for EU with Index 1, about 10% with Index 2, as in

[43] Appendix A.2 provides further intuitions.

[44] The corresponding smaller effect on RER thus stems from that the relative wage in the nontradable sector is positively related to the relative price of nontradables, which in turn affects the price index positively. Recall the discussion in section 4.1.

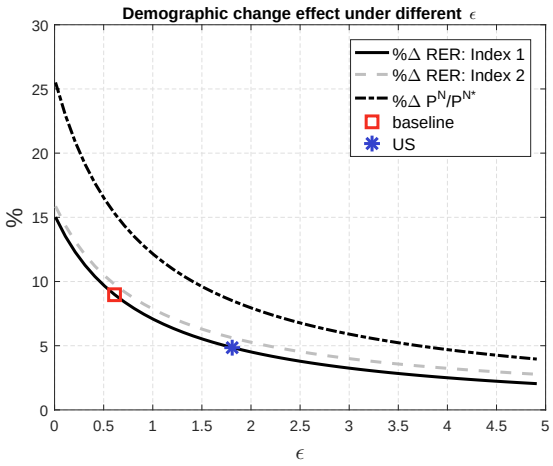


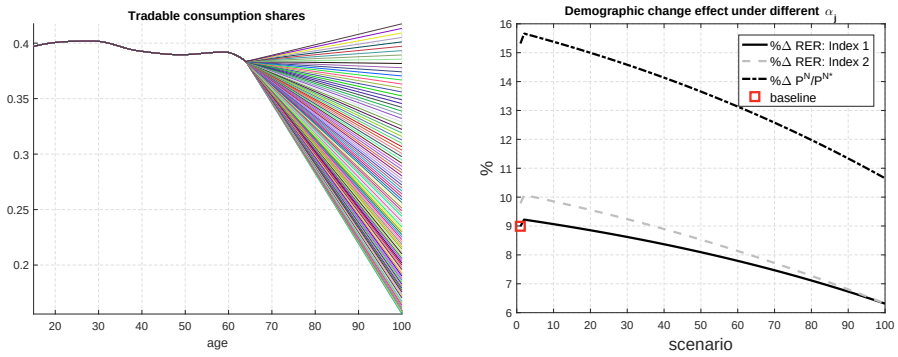
Figure 4: Demographic change effect: different labor market mobility (ϵ)

Note. Computed in partial equilibrium i.e. real interest rate fixed at the initial steady state value, 0.0545 with no relative productivity changes. (4.4) and (4.5) define Index 1 and 2.

Table 3). The other scenarios are ordered in such a way that a higher scenario index means that the tradable consumption share is on a higher path in Figure 5, panel (a).

Unsurprisingly, as can be seen in Figure 5 panel (b), when retirees consume relatively less nontradables the effect of demographic change on both RER is smaller. In the last scenario when workers consume about the same as retirees $\%\Delta RER$ is about 3 percentage points smaller than in the baseline. However, even when both workers and retirees consume about the same share of nontradables, the demographic change still leads to a significant real appreciation for the country that ages the most (EU) suggesting that aging matters for RER, not only because of the age-variant consumption composition but also because of the effect on savings and hence on the overall consumption demand in steady state which leads to price level differentials between the two countries aging differently (at least in partial equilibrium, see 'savings channel' in section 4.1).

The role of general equilibrium forces. The sensitivity analyses above have been conducted in partial equilibrium, namely by keeping the real interest rate in the final steady state at its endogenous value in the initial steady state. What happens when the real interest rate is allowed to be determined endogenously in the final steady state, i.e. in general equilibrium? We answer this question not only for



(a) (b)

Figure 5: Demographic change effect: different consumption shares in tradables

Note. Solution values are computed in partial equilibrium (i.e. real interest rate fixed at the initial steady state value, $r = 0.0545$) with no relative productivity changes. (4.4) and (4.5) define Index 1 and 2.

different degrees of inter-sectoral labor rigidity (ϵ) but also for different degrees of pension system generosity (i.e. different values of the net replacement rate, $\bar{d}$) in the two countries. Contrary to variations of ϵ, variations of $\bar{d}$ affect the real interest rate by affecting the savings decision. A more generous pension system (higher value of $\bar{d}$) crowds out private savings, thus tending to determine a higher value of the real interest rate in equilibrium (depending on the size of the country in the world economy).

Figure 6 shows the long-run impact of demographic change on RER (an increase means appreciation for EU) in general equilibrium for different values of ϵ and for different values of $\bar{d}$ for EU in the final steady state holding fixed the pension generosity for the US at the initial steady state value.[45] In the initial steady state $\bar{d}$ is equal to 0.45 in the US and to 0.6 in EU to be roughly consistent with the reported historical discrepancy between the European countries and the US (see Horlick (1970), Aldrich (1982)). It is apparent that, for given value of ϵ, the more the EU decreases the generosity of the pension system in the long-run the more its RER appreciates. The reason is that by decreasing the pension generosity there is less crowing-out effect on private savings. Therefore, with higher level of savings,

[45] In each simulation ϵ is equal between countries and it is the same in the initial versus the final steady state

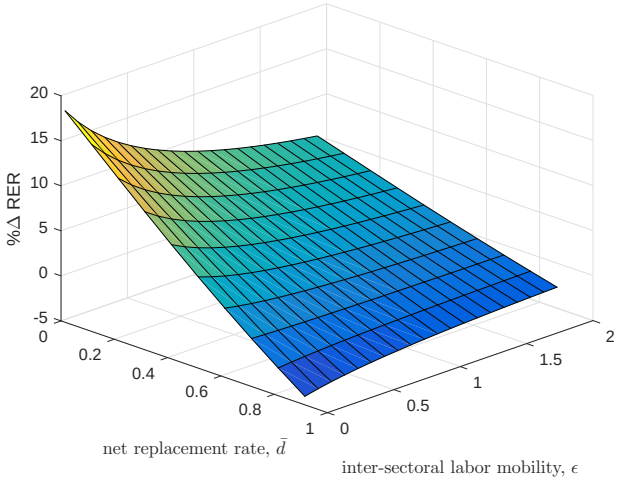


Figure 6: Demographic change effect: general equilibrium, different labor market mobility (ϵ) and pension generosity ($\bar{d}$).

Note. General equilibrium: real interest rate endogenous in the final steady state. $\%\Delta RER$ refers to the percentage variation of the RER (an increase means appreciation for EU) based on Index 1 as of (4.4). In the initial steady state the net replacement rate ($\bar{d}$) is 0.6 for EU, 0.45 for the US.

individuals can afford a higher level of consumption in the long-run which is associated with a more appreciated RER (the 'savings channel' explained in section 4.1). This occurs in general equilibrium: since only one country (EU) reduces the pension generosity the associated increase in savings at the world level does not generate a sufficient decrease of the real interest rate to nullify the positive effect on RER of the savings channel.

Overall, the RER appreciation for the country aging more survives in general equilibrium for most values of the net-replacement rate and, as expected, the effect is bigger the higher the degree of inter-sectoral labor rigidity (i.e. the lower ϵ).

8 Theoretical and empirical lesson on the relevance of demographics

The cointegration relationship estimated in section 2.2 between the REER and the set of fundamentals in each year needs to be interpreted as the long-run value to which the REER is predicted to revert once short-run fluctuations are over.

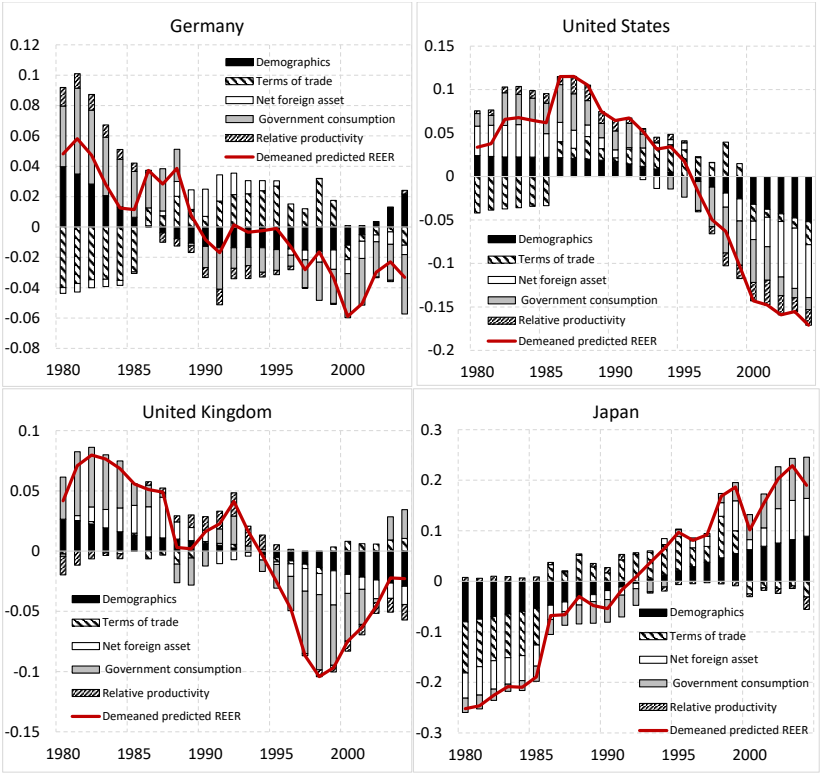


Figure 7: Predicted REER: the role of fundamentals (demeaned values)

Note. For each country i and for each fundamental k in year t, x_{it}^k, the predicted demeaned contribution value is $\beta^k x_{it}^k - (1/T)\sum_t \beta^k x_{it}^k$ where $T = 2004 - 1980 + 1$. In the figure we report the results when β^k is estimated with DOLS technique (see column (4) of Table 1).

To quantify the contribution of each fundamental, Figure 7 plots the long-run REER predicted by the DOLS estimation for a selection of advanced economies, demeaned to abstract from the contribution of the intercepts, together with each

fundamental's contribution. Demographics play a relevant role in these countries with magnitude comparable to government consumption, terms of trade and net foreign assets. For Japan, which ages the most in the sample, demographics play a more important role than in other countries. The relative productivity of the tradable sector which captures the "Balassa-Samuelson effect" seems to play a negligible role.

Figure 8: Predicted REER: mean % contribution to absolute deviation

Note. For each country and each year we compute the absolute deviation of predicted REER and evaluate the percentage contribution of each fundamental. We then average for each country over the sample period and finally across all countries for each of the *Advanced countries* and of the *Emerging markets economies*. See 20 for country decomposition

Considering for each country the percentage contribution of each factor to the absolute deviation of REER from its mean, then averaging over the sample period and across countries, demographics account for about 15% for advanced economies (government consumption: 29%, terms of trade: 28%, net foreign asset: 16%, relative productivity: 11%). For emerging markets the contribution of demographics is significantly smaller, at about 6%, while terms of trade matter more. Figure 8 reports these values while Figure 20 in Appendix reports the specific results for each of the 45 countries in the sample.

Our empirical results with DOLS estimation indicate that an increase of 1% of

the old dependency ratio of a country relative to its trading partners is associated with an appreciation of its REER by 0.29%. In order to relate this number to the quantitative prediction of the theoretical model, consider the following computation. The old-dependency ratio increases by 3.45 times in EU and by 2.28 times in the US from 1960 to 2050. Hence the EU ages about 50% more than US (its only trading partner in the model) according to this metric. Over the same period our model predicts that aging induces a RER appreciation for the EU *vs* US of about 10-11% in partial equilibrium and about 4-5% in general equilibrium (see the results in Table 3). Therefore our theoretical quantitative prediction is that a 1% increase of the old-dependency ratio relative to trading partners generates about 0.2% RER appreciation in partial equilibrium, and about 0.1% in general equilibrium. Notice that the empirical slope coefficient of 0.29% could be generated by the model in general equilibrium under a calibration that features a less generous pension system in Europe in the long-run as illustrated in section 7.[46]

To give a perspective, consider that the annual (compound) growth rate of the old-dependency ratio is about 1.5% for Europe, about 1% for the US. With a theoretical slope coefficient on the relative price of nontradables of about 0.3% in partial equilibrium (0.15% in general equilibrium), this means that every year nontradable goods in Europe become more expensive than in the US by about 0.15% in partial equilibrium (0.075% in general equilibrium) due to the stronger aging of the population. In terms of general price levels, this means 0.1% (0.05%) more expensive in Europe as compared to the US due to aging as nontradables account for about two-thirds of aggregate consumption. In other terms, if the US had always zero annual inflation, Europe would have 0.1% (0.05%). So this effect seems of plausible economic size, being neither trivial nor incredibly large.

[46] See Figure 6. When the inter-sectoral labor mobility ϵ and the long-run generosity of the EU pension system $\bar{d}$ are sufficiently small, the long-run impact of aging can be more than 15% RER appreciation for EU, namely a slope coefficient of more that 0.3% ($= 15/50$).

9 Concluding remarks

This paper analyzes theoretically and empirically the effect of demographic change on the RER. The increased demand for nontradable goods and services occurring with aging determines a RER appreciation for the economy that ages faster than its trading counterparts. This mechanism is for the first time quantified within a general equilibrium model. The appreciating force is due to the simultaneous presence of an aging-induced change in demand composition that favors the relative consumption of nontradables and imperfect substitutability of hours worked across sectors. The magnitude of the model-based result depends on whether we account for changing relative productivity across sectors and the general equilibrium effect via aging lowering the world interest rate. Our empirical analysis points in the same direction and order of magnitude of the effect of aging on RER. A country faces a RER appreciation of about 0.2% according to the model in partial equilibrium (0.1% in general equilibrium), 0.29% according to empirics, when it faces an increase of 1% in the number of elderly over the number of people in the working-age relative to its trading partners. These numbers attribute a non-negligible role to demographic change which in our panel cointegration analysis explains about 15% of the mean absolute deviation of the REER movements associated with fundamentals for advanced economies. The economic significance of this effect calls for considering demographics as a fundamental determinant of long-run RERs.

Looking forward, given the current demographic projections, it is likely that demographics will have more prominence in explaining cross-country price differentials. To what extent this will occur will crucially depend on how aging and other forces in the economies affect the world real interest rate. Aside from the relevant topic on how aging will affect real interest rate, our analysis opens also other avenues for future research. Two of them seem particularly important to us. First, how differences across countries in government interventions, via not only different pension schemes but also different types of welfare assistance to the elderly, might foster or attenuate the impact of the old-age consumption propensity towards nontradables on relative prices. Second, how the shift towards aging societies and the progressive reallocation of resources from the tradable sector to the nontradable

sector might endogenously affect aggregate productivity. We hope our contribution can encourage research in these directions.

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A Model appendix

A.1 Stationary equilibrium: optimality conditions

In a two-countries world, for a numerical solution of this system one has to guess r, P^N, P^{N*} and verify that they satisfy the clearing conditions. Suppose that the economy is in steady state in a particular year, say 1960. Then, following Domeij and Flodén (2006), we take the number of people in each age-class in 1960 from the data and compute the implied conditional survival probabilities for each age $j \in \{1, 2, \cdots J\}$ with $N_{-1} = N_0$ such that $s_0 = 1$:

$$s_j = \frac{N_j}{N_{j-1}} \tag{A.1}$$

from which $\pi_j = \prod_{k=0}^{j} s_k$ can be found. From the firms' maximization problem:

$$w^T = (1-\psi^T) Z^T \left(\frac{r+\delta}{\psi^T} \right)^{-\frac{\psi^T}{1-\psi^T}} \tag{A.2}$$

$$w^N = P^N (1-\psi^N) Z^N \left(\frac{r+\delta}{\psi^N P^N} \right)^{-\frac{\psi^N}{1-\psi^N}} \tag{A.3}$$

Given that hours worked are aggregated via a CES function, the hourly wage rate in the economy is:

$$w = \left[\theta (w^T)^{\varepsilon+1} + (1-\theta)(w^N)^{\varepsilon+1} \right]^{\frac{1}{\varepsilon+1}} \tag{A.4}$$

Pension and tax rate are given with little derivation:

$$L = \sum_{j=0}^{j_r-1} h_j N_j \tag{A.5}$$

$$\bar{h} = \frac{\sum_{j=0}^{j_r-1} h_j}{j_r} \tag{A.6}$$

$$d = \overline{d} w (1-\tau) \bar{h} \tag{A.7}$$

$$\tau = \frac{\overline{d}\bar{h} \sum_{j=j_r}^{J} N_j}{L + \overline{d}\bar{h} \sum_{j=j_r}^{J} N_j} \tag{A.8}$$

Income for all j is given by:

$$y_j = (1-\tau) w h_j I(j < j_r) + dI(j \geq j_r) \tag{A.9}$$

By solving recursively the household's budget constraint one gets the following equation for individual consumption in T at age 0:

$$c_0^T = \frac{\alpha_0 \sum_{j=0}^{J} \left(\frac{1}{1+r}\right)^j \pi_j y_j}{\sum_{j=0}^{J} \beta^j \pi_j + \pi_{jb} \Gamma \beta^{jb} \left(1 - (1+r)^{jb+1}\right)} \tag{A.10}$$

that, given the standard intra- and inter-temporal conditions for consumption, allows to compute for all j:

$$c_j^T = \frac{\alpha_j}{\alpha_0} \beta^j (1+r)^j c_0^T \tag{A.11}$$

$$c_j^N = \frac{1-\alpha_j}{\alpha_j} \frac{c_j^T}{P^N} \tag{A.12}$$

Given that the aggregate hours worked, h_j, are exogenously given, the sectoral hours worked are (recall that from $j = j_r$ the hours worked are assumed to be zero):

$$h_j^T = \theta h_j \left(\frac{w^T}{w}\right)^{\varepsilon} \tag{A.13}$$

$$h_j^N = (1-\theta) h_j \left(\frac{w^N}{w}\right)^{\varepsilon} \tag{A.14}$$

Utility maximization leads to the following bequest that is split equally among the new entrants in the market in the form of initial assets:

$$b_{j_b} = \frac{c_0^T}{\alpha_0}\Gamma\beta^{j_b}(1+r)^{j_b} \tag{A.15}$$

$$a_0 = \frac{b_{j_b}N_{j_b}}{N_0} \tag{A.16}$$

Directly from the household's budget constraint, the asset holdings for all j (with perfect annuities market):

$$a_{j+1} = \frac{a_j(1+r)}{s_j} - c_j^T - P^N c_j^N - b_{j_b}I(j=j_b) + y_j \tag{A.17}$$

From the clearing conditions and firms' maximization problem:

$$L^T = \sum_{j=0}^{J} h_j^T N_j \tag{A.18}$$

$$L^N = \sum_{j=0}^{J} h_j^N N_j \tag{A.19}$$

$$K^T = Z^T L^T \left(\frac{r+\delta}{\psi^T}\right)^{-\frac{1}{1-\psi^T}} \tag{A.20}$$

$$K^N = Z^N L^N \left(\frac{r+\delta}{P^N\psi^N}\right)^{-\frac{1}{1-\psi^N}} \tag{A.21}$$

The same set of equations shall also hold for the rest-of-the-world denoted by a "$*$". Then, the unknown P^N, P^{N*}, r are determined by solving the following system in three equations and three unknowns:

$$(K_t^N)^{\psi^N}(Z_t^N L_t^N)^{1-\psi^N} = \sum_{j=0}^{J} N_{t,j}c_{t,j}^N \tag{A.22}$$

$$(K_t^{N*})^{\psi^{N*}}(Z_t^{N*} L_t^{N*})^{1-\psi^{N*}} = \sum_{j=0}^{J} N_{t,j}^* c_{t,j}^{N*} \tag{A.23}$$

$$K^T + K^N + K^{T*} + K^{N*} = \sum_{j=0}^{J} a_{j+1}N_j + \sum_{j=0}^{J} a_{j+1}^* N_j^* + a_0 N_0 + a_0^* N_0^* \tag{A.24}$$

A.2 On the role of imperfect substitutability of labor between sectors

Consider the household's CES aggregator for the hours worked in the two sectors (age and time subscripts are suppressed for brevity):

$$h = \left[\theta^{-\frac{1}{\epsilon}}(h^T)^{\frac{\epsilon+1}{\epsilon}} + (1-\theta)^{-\frac{1}{\epsilon}}(h^N)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{A.25}$$

To visualize the implications of this assumption, Figure 9 depicts the set of optimal feasible hours worked in the two sectors ('transformation possibilities frontier', left panel) and the associated relative wage (the slope of the frontier, right panel) for different values of ϵ (the degree of labor mobility between sectors). The left panel shows the possible combinations of sectoral hours to have a given number of total hours $\overline{h}$ normalized to 1 in case of sectoral specialization ($\overline{h} = \theta^{-1/(\epsilon+1)}$) with the additional assumption of symmetry ($\theta = 0.5$).[47] The dotted lines represent the frontier in the case of perfect labor mobility, i.e. the function: $h^T = \overline{h} - h^N$ when $\epsilon \to \infty$. The perfect labor mobility frontier is tangent to the imperfect labor mobility frontier at the point where total hours are maximized which, due to the assumed symmetry, corresponds to hours being equal between sectors ($h^N = h^T$). Consider to be at one of those tangent points where the relative wage is 1. To hire relatively more hours in the nontradable sector (given the exogenous $\overline{h}$) there is no need of relative wage adjustment under the case of perfect labor mobility. However, when $\epsilon < \infty$, there needs to be a permanent increase of the relative wage of nontradables to make the household willing to supply that amount of relative hours according to the preferences as represented by the frontier in the left-panel. The increase is higher the higher the degree of labor immobility i.e. the lower ϵ.

One can see the horizontal distance between the imperfect mobility frontier and the perfect mobility frontier as the number of hours in nontradables that are

[47] The left panel of Figure 9 plots the following function:

$$h^T = \left(\frac{(\overline{h})^{\frac{\epsilon+1}{\epsilon}} - (1-\theta)^{-\frac{1}{\epsilon}}(h^N)^{\frac{\epsilon+1}{\epsilon}}}{\theta^{-\frac{1}{\epsilon}}}\right)^{\frac{\epsilon}{\epsilon+1}}$$

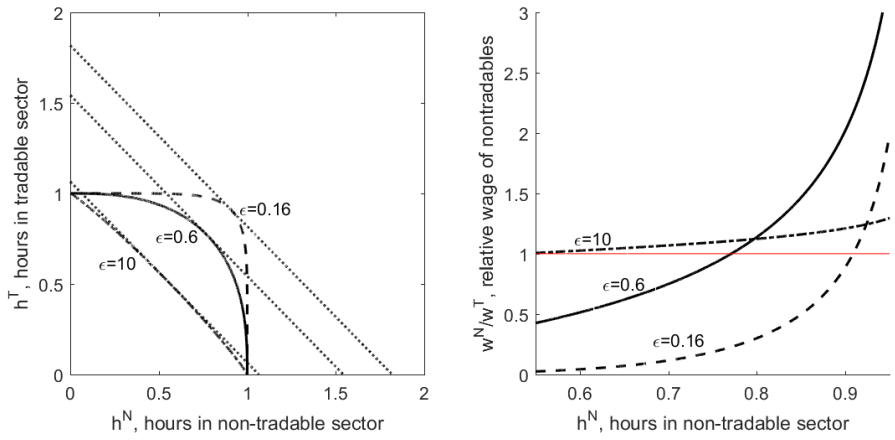

Figure 9: Hours worked in the the two sectors and the associated relative wage

Note. Computation based on the household's CES aggregator in (4.3).

forgone, for given number of hours in tradables, due to that the household has a certain preference to work in both sectors.

A.3 Two-period model

To substantiate the discussion in section 4.1 we calibrate the two-period model comparing EU ans US using demographic data from UN (2017) and targeting values similarly to the calibration procedure used for the full age-structure model employed in section 5. To isolate the impact of demographic change we assume that the two countries are in all equal except for demographics. We provide a graphical analysis based on equations (4.6) and (4.7) in the main text. These equations are derived as follows. Consider the optimal conditions of section A.1 when the representative household is alive for two periods: age $j = 0$ as a worker, age $j = 1$ as retiree. In this case the conditional survival probability of being alive from period 0 to 1 is the old-dependency ratio: $s_1 = n_1/n_0$ denoting the number of workers and retirees by n_0 and n_1 respectively.

The relative supply of labor is immediately found by combining (A.13) and

(A.14):

$$\frac{h^N}{h^T} = \frac{1-\theta}{\theta}\left(\frac{w^N}{w^T}\right)^{\epsilon} \tag{A.26}$$

The goal now is to derive the relative demand of labor taking into account all the other optimal conditions, for given real interest rate r. To this end, the clearing condition of the market for non-tradable goods is essential (see (A.22)):

$$(K^N)^{\psi^N}(Z^N L^N)^{1-\psi^N} = n_0 c_0^N + n_1 c_1^N$$

By plugging (A.21), (A.3), (A.19) in the LHS, and (A.11), (A.12) in the RHS:

$$\frac{n_0 h^N w^N}{P^N(1-\psi^N)} = \frac{c_0^T}{\alpha_0 P^N}\left[n_0(1-\alpha_0) + n_1(1-\alpha_1)\beta(1+r)\right]$$

that can be easily rewritten as (using $s_1 = n_1/n_0$):

$$\frac{h^N w^N}{(1-\psi^N)} = \frac{c_0^T}{\alpha_0}\left[(1-\alpha_0) + (1-\alpha_1)s_1\beta(1+r)\right] \tag{A.27}$$

The expression for c_0^T/α_0 is given by (A.10) which for the two-period case reads:

$$\frac{c_0^T}{\alpha_0} = \frac{y_0 + \frac{s_1 y_1}{1+r}}{1 + s_1\beta\left[1 + \Gamma(1-(1+r)^2)\right]}$$

Income is given by (A.9):

$$y_0 = (1-\tau)(w^T h^T + w^N h^N), \quad y_1 = d = \frac{\tau(w^T h^T + w^N h^N)}{s_1}$$

where from (A.5) to (A.8) the tax rate is given by (normalizing the efficiency of labor: $h_0 = 1$):

$$\tau = \frac{\bar{d}s_1}{1+\bar{d}s_1} \tag{A.28}$$

Therefore:

$$\frac{c_0^T}{\alpha_0} = \frac{\left(w^T h^T + w^N h^N\right)\left(1 - \tau\frac{r}{1+r}\right)}{1 + s_1\beta\left[1 + \Gamma(1-(1+r)^2)\right]}$$

Plug the last expression into (A.27) to have:

$$\frac{1 + s_1\beta\left[1 + \Gamma(1-(1+r)^2)\right]}{\left[(1-\alpha_0) + (1-\alpha_1)s_1\beta(1+r)\right]} = \left(\frac{w^T h^T}{w^N h^N} + 1\right)\left(1 - \tau\frac{r}{1+r}\right)(1-\psi^N)$$

i.e.

$$\frac{1 + s_1\beta\left[1 + \Gamma(1-(1+r)^2)\right]}{\left[(1-\alpha_0) + (1-\alpha_1)s_1\beta(1+r)\right]\left(1 - \tau\frac{r}{1+r}\right)(1-\psi^N)} - 1 = \frac{w^T h^T}{w^N h^N} \tag{A.29}$$

which identifies the relative demand of labor. By equating (A.26) and (A.29) one finds the equilibrium relative wage w^N/w^T. Then, by combining (A.2) with (A.3) the relative price of non-tradable goods is easily pinned down by:

$$P^N = \left[\frac{w^N}{w^T}\frac{Z^T}{Z^N}\frac{1-\psi^T}{1-\psi^N}\frac{\left(\frac{\psi^T}{r+\delta}\right)^{\frac{\psi^T}{1-\psi^T}}}{\left(\frac{\psi^N}{r+\delta}\right)^{\frac{\psi^N}{1-\psi^N}}}\right]^{1-\psi^N} \tag{A.30}$$

Table 4 summarizes the choice of the parameter values. A period in the model corresponds to $p = 40$ years. The capital share of income is set to the standard value of about 1/3, assumed to be the same between sectors and countries ($\psi^T = \psi^N = \psi^{T*} = \psi^{N*}$) so that also the capital-output ratio is the same between sectors and countries. Targeting a yearly capital-output ratio of 2.57 and an investment-output rate of 0.2 implies an yearly depreciation of capital of about 7.78% which over a span of p years implies a depreciation in the model of $\delta = 1 - (1 - 0.0778)^p$. The net replacement rate which corresponds to the percentage of the average working earnings provided as pension benefit is set to 45%. We abstract from bequest motive so that $\Gamma = 0$. The weight to tradable labor in the CES aggregator is set to about the mid-point value between EU and US of the average tradable content of labor compensation according to EUKLEMS data over the period 1970-2007. The degree of labor mobility ϵ is set to 0.61 which corresponds to the "whole sample" estimate

provided by Cardi and Restout (2015). The age-variant share of consumption is set to 39.5% for young cohorts and to 27% for old-cohorts according to our estimates on US private consumption data (see Figure 1). The relative labor-augmenting technology Z^T/Z^N is such that firms in the tradable sector have a multiplicative factor 50% higher than in the nontradable sector, which is in line with what assumed by Cardi and Restout (2015) for a representative OECD economy. The individual discount factor β is such that the initial steady state delivers an annualized real interest rate of 5.45% which is the value implied by targeted annual capital- and investment-output ratio of 2.57 and 0.20 respectively.

Table 4: TWO-PERIOD MODEL: PARAMETER VALUES (INITIAL STEADY STATE)

Parameter	EU	US	Note
p	40		number of years corresponding to 1 period
$\psi^T = \psi^N$	0.34		output elasticity to capital, both sectors
δ	$1-(1-0.0778)^p$		depreciation rate of capital (target: $K/Y = 2.57$, $I/Y = 0.2$, source: WDI)
$\bar{d}$	0.45		net replacement rate, US 2014, source: OECD
Γ	0		preference for bequest
θ	0.35		tradable content of labor compensation, average 1970-2007, source: EUKLEMS
ϵ	0.61		degree of labor mobility, 'whole sample' in Cardi and Restout (2015)
α_0	0.395		young-cohort tradable share of consumption, age 15-64 US average, see Figure 1
α_1	0.27		old-cohort tradable share of consumption, age 65+ US average, see Figure 1
Z^T/Z^N	1.5		relative labor-augmenting technology, source Cardi and Restout (2015)
β	0.9827^p		individual discount factor, to target $r = [1+\psi/(K/Y)-(I/Y)/(K/Y)]^p - 1$
s_1	0.1636	0.1593	1960 old-dependency ratio: 65+/15-64, source: UN (2017)

Table 5 reports the resulting percentage change in the RER (RER), defined as the price index in Europe over the price index in the United States, under the different scenarios.

Table 5: Initial *vs* final steady state: percentage change of $RER = P^{EU}/P^{US}$

	$\%\Delta RER$	
	PE ($r = 5.45\%$)	GE ($r = 3.74\%$)
$\alpha_1 < \alpha_0$	87.4%	6.24%
$\alpha_1 = \alpha_0$	19.14%	4.26%

While these numbers are only indicative given the simplicity of the model, they make clear that the region aging the most (Europe) faces a RER appreciation which is: *(a)* significantly dampened when general equilibrium effects induced by aging

are considered; *(b)* mainly due to the fact that the elderly have a consumption bias towards nontradable goods.

Given a fixed replacement rate of 45% in both regions, the labor income tax rate τ is projected to increase from about 7% in both regions in 1960 to about 20% in Europe and 14% in the United States in 2050 (to make the government budget balanced in each period, see equation (A.8)).[48] This evolution underlies the results discussed above. However, as long as the replacement rate is the same across countries the different evolution of the tax rate due to the different aging pattern is irrelevant for the long-run change of the RER. This occurs despite the initial and final level of both the real interest rate and relative prices gets significantly changed.[49]

For given individual discount factor β, when there is no pension system ($\tau = 0$) the real interest rate is lower in both the initial and final steady state (4.96% and 2.65% respectively) compared to the case of PAYGO pension system. The reason is that with no pension system private savings are not crowded out. Therefore, for the pension system to matter for the change in relative prices one should observe a significant change of the pension systems *across countries* , i.e. a significant change of the replacement rate for the case analyzed here.

For example, in the context of this model, we would see that the country aging the most faces a RER appreciation in the long-run if it reduces the replacement rate while its trading partners do not. This would increase its saving rate compared to the trading partners and so its aggregate level of consumption in the long run which would inflate the relative price of nontradables, more so if there is a old-age consumption bias towards nontradables. Of course, this would occur if the general equilibrium effects were not strong enough to counteract this inflationary pressure.[50] But potential long-run differences of the pension systems across coun-

[48] The tax rate is $\tau = \bar{d}s_1/(1 + \bar{d}s_1)$, evidently an increasing function of the old-dependency ratio s_1.

[49] Our simulations reveal that the $\%\Delta RER$ for the case of no pension systems in both countries ($\tau = 0$) is about 6% in general equilibrium. Hence irrelevant. Compare it with the 6.24% in Table 5

[50] Notice that the real interest rate decreases more if there are no pension systems: it decreases about 2.3 percentage points (4.96-2.65) instead of the 1.7 percentage points of the baseline case with pension systems (5.45-3.74).

tries, highly unpredictable and hard to detect, go beyond the scope of our current analysis.

A.4 The role of country specific parameters

Table 6: COUNTRY-SPECIFIC CALIBRATION: INITIAL STEADY STATE

Parameter	EU	US	Note
$\psi^T = \psi^N$	0.34	0.34	output elasticity to capital (Cobb-Douglas production function, both sectors)
δ	0.0778	0.0778	depreciation rate of capital (target: $K/Y = 2.57$; $I/Y = .2$. Source: WDI)
$\overline{d}$	0.45	0.45	net replacement rate: % of average net working earnings, US 2014, source: OECD
Γ	0	0	preference parameter for bequest
θ	0.4	0.3	tradable content of labor compensation, average 1970-2007. Source: EUKLEMS
ϵ	0.89	1.8	degree of labor mobility (immobility: $\epsilon = 0$). Source: Cardi and Restout (2015)
J	86	86	terminal life-period (age 100, still alive)
jr	50	50	last period in working-life (age 64), see Carvalho et al. (2016) Tab. 2
N_j	Figure 17	Figure 17	number of people in each age-group j. Source: UN (2017)
h_j	Figure 19	Figure 19	labor supply in efficiency units. Source: Domeij and Flodén (2006)
α_j	Figure 1	Figure 1	tradable shares of consumption. Sources: EUROSTAT 2010, CES 2015
β	0.9652	0.9652	individual discount factor
Z^T/Z^N	1.5768	1.5001	relative labor productivity (target: $P^N = 1.0435$, $P^{N*} = 1.0574$, 1970 EUKLEMS)

Note. See section 7.

Table 6 reports the parameter values for the county-specific calibration. The differences with the baseline calibration (see Table 2) are the following. First, we assume that θ differs across countries according to the average tradable content of labor compensation found in EUKLEMS data over the period 1970-2007. Second, the degree of labor mobility between sectors is set to the country-specific value provided by Cardi and Restout (2015).[51] Third, the age-varying consumption shares are allowed to differ across countries according to our estimates (see Figure 1). Fourth, the parameter capturing the relative labor augmenting technology, Z^T/Z^N, is endogenous in the initial steady state. Specifically, for both countries we target the respective relative price of nontradables in 1970 (earliest available EUKLEMS data) for the initial steady state: $P^N = 1.0435$, $P^{N*} = 1.0574$. Consequently,

[51] They provide estimates for 14 OECD countries, of which only 8 are EA12 members (Belgium, Finland, France, Germany, Ireland, Italy, Netherlands, Spain). Hence, the final value of ϵ for EA12 is the GDP-weighted average of these 8 countries. Weights are obtained from the 2000 "Gross domestic product at market prices, chain linked volumes (2005), million euro" provided by EUROSTAT. Comparing the value obtained for EA12, 0.89, with the one Cardi and Restout (2015) report for United States, 1.8, it implies that US has more labor mobility between sectors than the EA. However both have higher labor mobility than what estimated using the whole sample of OECD economies which we used for both countries in our baseline calibration.

the initial steady state has the demographics of 1970 (not 1960 as in the baseline calibration).

Given all parameters in Table 6 except β and Z^T/Z^N, we solve for β and Z^T/Z^N such that the capital market clearing condition at the world level and the nontradable goods clearing condition at the country level are satisfied which gives: $\beta = 0.9652, \quad \frac{Z^T}{Z^N} = 1.5768, \quad \frac{Z^{T*}}{Z^{N*}} = 1.5001$ obtained by setting $Z^N = Z^{N*} = 1$. Notice that the relative labor-augmenting technology Z^T/Z^N values are similar in the two countries and close to 1.5 as used for both countries in the baseline calibration.

Table 7 presents simulation results with the country-specific calibration under the same scenarios as the baseline analysis (see Table 3) again assuming that the final steady state is year 2050. Compared with the baseline calibration, the absolute long-run impact of demographic change on the RER is dampened for three main reasons. First, we consider year 1970 instead of 1960 as starting point. This implies that the relative aging process is less dramatic.[52] Second, according to the country-specific estimates the degree of labor mobility ϵ is higher in both countries compared to the whole sample estimate used in the baseline calibration. As discussed above, higher labor mobility flattens the relative supply of nontradable goods so that any relative demand shifts has less impact on the relative price. Third, using the EU age-varying consumption shares dampens both the overall level of nontradable consumption in the economy and the one related to old cohorts (compare the thick lines in Figure 1).

Nonetheless, the general effect of aging remains in the same order of magnitude. Consider the following. Under the country-specific calibration the biggest effect predicted by the model is about 4% RER appreciation for EU from 1970 to 2050. According to the old-dependency ratio, EU ages about 34% ($\approx 2.9/2.16 - 1$) more than US. This means that a 1% increase of the relative old-dependency ratio is associated with about 0.12% ($\approx 4/34$) RER appreciation. The corresponding number is 0.2% for the baseline calibration results.[53]

[52] From 1970 (1960) the old-dependency ratio becomes 2.9 (3.45) times higher in EU, 2.16 (2.28) times higher in US. See Figure 16.

[53] See section 8.

Table 7: INITIAL (1970) *vs* FINAL (2050) STEADY STATE: EFFECT OF DEMOGRAPHIC CHANGE

		Initial steady state	Final steady state		
			Demographics	**Relative productivity**	**Dem. + Str. change**
		General Equilibrium		***General Equilibrium***	
P^N	**EU**	1.0435	1.0570	2.9303	2.9682
	US	1.0574	1.0496	2.9693	2.9473
r	**world**	0.0545	0.0428	0.0545	0.0428
$RER = \frac{P^{EU}}{P^{US}}$	**Index 1 (eq. 4.4)**	1.0145	1.0345	0.9114	0.9282
	Index 2 (eq. 4.5)	0.9875	0.9986	0.8870	0.8952
Demographics impact on RER	**Index 1 (eq. 4.4)**		1.97%		1.84%
	Index 2 (eq. 4.5)		1.12%		0.92%
			Partial Equilibrium		
P^N	**EU**		1.2788	2.9303	3.5910
	US		1.1908	2.9693	3.3440
r	**world**		0.0545	0.0545	0.0545
$RER = \frac{P^{EU}}{P^{US}}$	**Index 1 (eq. 4.4)**		1.0557	0.9114	0.9472
	Index 2 (eq. 4.5)		1.0189	0.8870	0.9135
Demographics impact on RER	**Index 1 (eq. 4.4)**		4.06%		3.93%
	Index 2 (eq. 4.5)		3.18%		2.99%

Note. Country-specific calibration of Table 6. See (4.4) and (4.5) for definition of Index 1 and Index 2.

B Data pattern and calibration appedix

B.1 Age-dependent sectoral consumption shares

A key element of our analysis is the age-dependent consumption shares depicted in Figure 1. To build those shares we have used data on aggregate consumption expenditures by age on broad categories which are roughly comparable between EU countries and US despite different data sources: EUROSTAT for the former, Consumer Expenditure Survey (CEX) for the latter. using CEX data which provides more details as compared to EUROSTAT data we identify which consumption categories that drive the results in Figure 1.

Figure 10 and 11 provide an answer. At older ages households decrease the share of consumption expenditure on most categories in favor of "health-care" and "housing" (excluding furnishing and equipment). In particular, "Transportation" is the category that faces the biggest decrease on the tradable side. Compared to those aged between 55 and 64, those aged 75 and more devote about 6 percent-

	Disaggregated Consumption Categories	Under 25 years (1)	25-34 years (2)	35-44 years (3)	45-54 years (4)	55-64 years (5)	65-74 years (6)	75 years and older (7)
TRADABLE	Food at home	9.1	8.2	8.9	8.1	8.8	8.7	8.4
	Alcoholic beverages	0.9	1.2	1.3	0.9	1.2	1.0	0.7
	furnishings and equipment	3.7	3.9	3.8	3.6	4.3	4.2	3.2
	Apparel and services	4.5	4.3	4.7	4.5	3.3	3.1	2.1
	Transportation	21.1	21.8	20.1	20.9	20.7	19.0	15.6
	Tobacco products and smoking supplies	0.8	0.8	0.7	0.8	0.8	0.6	0.4
NON-TRADABLE	Food away from home	8.3	7.1	7.0	6.4	5.9	5.5	5.0
	Housing minus furnishings and equipments	34.0	36.9	36.9	34.1	33.4	34.8	39.2
	Healthcare	3.1	6.2	7.1	8.3	10.6	13.6	17.4
	Entertainment	4.6	5.6	5.8	5.9	6.9	7.1	5.2
	Personal care products and services	1.4	1.4	1.5	1.4	1.5	1.4	1.6
	Reading	0.1	0.2	0.2	0.2	0.2	0.4	0.6
	Education	8.4	2.5	2.2	4.7	2.4	0.7	0.7
	Total	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Figure 10: Sectoral consumption shares by age, US 2015

Note. Data source: "2015 Aggregate Expenditure Shares Tables, Age of reference person" available at https://www.bls.gov/cex/csxashar.htm where the reader is redirected for the complete list of subcategories. The reference person is defined as "The first member mentioned by the respondent when asked to 'Start with the name of the person or one of the persons who owns or rents the home' ".

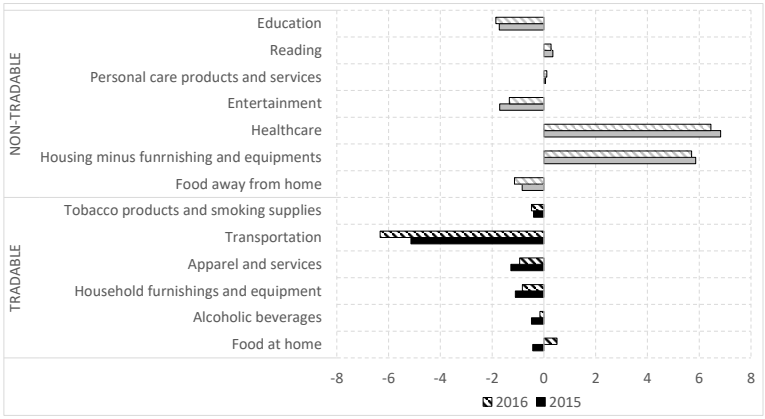

Figure 11: Percentage points variation between age 55-64 and 75+

Note. Numbers are the difference between column (7) and (5) in Figure 10.

age points more of their expenditure to both housing and health-care. Of notice is that "Transportation", "Housing" and "Health-care" are the categories that take the biggest shares in overall expenditure on average over all ages. Therefore, their change gives more relevance for the aggregate. For example, both tobacco products and alcoholic beverages decreases by more than 40 percent going from age 55-64 to 75+. But their impact is negligible compared to "housing minus furnish-

ing and equipment" which increases by less than 20 percent from age 55-64 to 75+ but takes a greater chunk of the overall expenditure basket. Health-care increases dramatically by 65 percent going from age 55-64 to 75+ so that about 17.4 percent of the expenditure basket of those aged 75 and more is devoted to health care (for households aged under 25 the figure is about 3.1 percent).
As Figure 11 shows, these numbers are not significantly different when one considers year 2016 instead of 2015 of the CEX survey.

B.1.1 Exploiting multiple cross-sectional data

To validate our results above we analyze the CEX database exploiting the multiple cross-sectional dimension (different US households cross-sections over time) – not only the aggregate static dimension as above. We do so going back in time to check whether what we found above is a relatively permanent feature and not an artifact driven by a unique set of very recent observations. Additionally, we use a more disaggregated classification than the one used above.

We use the data-set provided by Aguiar and Hurst (2013) which is complied from the National Bureau of Economic Research CEX extracts including all waves from 1980 to 2003. The survey is conducted every quarter but the data-set collapses the four interviews into a single annual observation per household.[54] The reader is redirected to their paper and web-pages for details. The final data-set we analyze differ from theirs in two dimensions. *(i)* Since their interest is in nondurable expenditure, they exclude some categories which instead we consider. Following their classification, these categories are grouped under "Health", "Education", "Durables and Household Maintenance" and "Extra" which together constitute about 25 percent of household annual monetary outlays adding 20 categories for a total of 49 (see Table 15). *(ii)* We consider households in which the head is between the ages of 25 and 85, thus extending their analyzed age-segment by 10 years.[55] Our resulting sample is composed by 58,467 households (which compares to 53,412 in their

[54] The data-set is publicly available here: http://www.markaguiar.com/aguiarhurst/lifecycle/datapage.html

[55] A head is defined as the member who identifies himself or herself as the "head of household" in the survey.

baseline analysis).

We follow closely the empirical methodology in Aguiar and Hurst (2013) where the life cycle impact is identified from cohort variation using the multiple cross-sections in the sample. We estimate the following regression:

$$s_{it}^k = \beta_0^k + \beta_{age}^k \text{Age}_{it} + \beta_t^k \text{D}_t + \beta_{ft}^k \text{Family}_{it} + \beta_c^k \text{Cohort}_{it} + \varepsilon_{it}^k \tag{B.1}$$

where s_{it}^k is the share of nominal expenditure of household i during year t devoted to consumption category k; Age_{it} is a vector of 60 1-year age dummies (for ages 26–85) referring to the age of the household head; D_{it} is a vector of normalized year dummies; Family_{it} is a vector of family structure dummies; Cohort_{it} is a vector of 1-year birth cohort dummies (1905-1968).[56] We consider two consumption categories, tradable and nontradable: $k \in \{\text{T}, \text{NT}\}$, according to the classification provided in Figure 15. Notice that here we classify each sub-category (not the macro-category like in section B.1) into either tradable or non-tradable. We thereby assess whether our empirical finding is robust to a more disaggregated classification.[57]

The coefficients on the constant β_0^k together with the age dummies, β_{age}^k, capture the impact of the life-cycle conditional on normalized year, family size and cohort fixed effects. Taken singularly, each of the age coefficients β_{age}^k represents the deviation of the consumption share on category k for those aged age from the corresponding share of those aged 25.

Figure 12 plots the estimated constant β_0^T plus the estimated coefficients β_{age}^T in (B.1) when the regressors are only the age and the year fixed effects, Age_{it} and

[56] See Aguiar and Hurst (2013). Like in their specification, Family_{it} comprises a marital status dummy, 10 household size dummies, and controls for both the number and age of household children aged 21 or under. D_t is a year dummy with the following two restrictions: (1) average zero over the sample (2) its covariance with a time trend is zero.

[57] In section B.1 Both "Housing" and "Transportation" have sub-categories whose classification into either tradadable or nontradable might be doubtful. For example, while within "Transportation" "vehicle purchases" are easily classifiable as tradable, less obvious is the classification of e.g. "Maintenance and Repairs" or "public and other transportation" which might include some local, hence nontradable, components. To account for this possibility we here classify some subcategories of "Transportation" as nontradable: "Repair, Greasing, Washing, Parking, Storage, Rental", "Bridge, Tunnel, Ferry, and Road Tolls", "Mass Transit Systems" (see Figure 15).

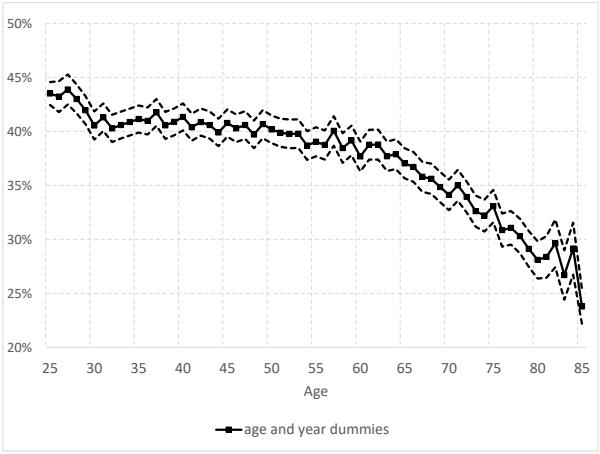

Figure 12: Consumption expenditure share on tradable categories

Note. Each point of the continuous line represents the estimated constant β_0^T plus the estimated coefficients β_{age}^T of an OLS regression on specification (B.1) excluding Family$_{it}$ and Cohort$_{it}$. The dashed lines around each point represent the regression coefficient plus and minus two times its standard errors.

D_{it}.[58] These results closely comparable to those of Figure 1 where additionally here we consider multiple cross-section of households over years 1980-2003 controlling for year fixed effects.

The resulting profile is remarkably similar to the one presented in Figure 1 both in level and in shape: households devote about 40 percent of their expenditure to tradable categories up until age 60; after that they decrease the share of expenditure on tradables reaching a level as low as 24 percent at age 85.

Figure 13 compares the life-cycle profile of the expenditure share on tradables of Figure 12 ("age and year") with specifications that additionally include family structure only and then both family structure and cohort fixed effects. The addition of family composition effect does not significantly alter the profile. However, when also birth cohort controls are added the profile is significantly steepened: the share of expenditure on tradables steadily decreases as age increases. The discrepancy between age 85 and 25 is about 40 percentage points, double compared to the "age

[58] Notice that the regressor Age$_{it}$ excludes age 25 so that the life cycle impact at age 25 is the constant β_0^T.

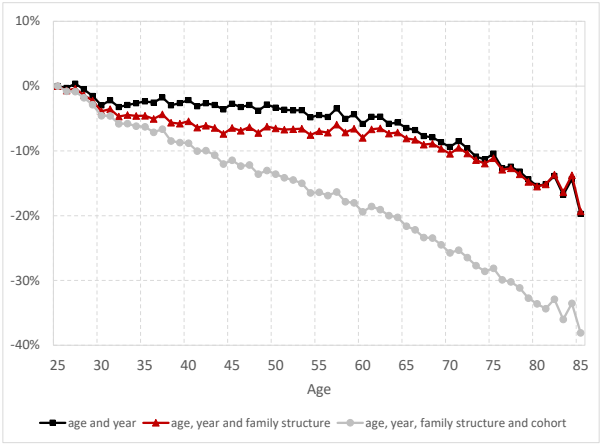

Figure 13: Consumption expenditure share on tradable categories, age 25 = 0

Note. The lines represent the estimated coefficient β^T_{age} from an OLS regression based on (B.1): "age and year" include only Age_{it} and D_t; "age, year and family structure" add Familiy_{it}; "age, year, family structure and cohort" add Cohort_{it}.

and year" specification. This result points to the non-negligibility of cohorts effects. Nonetheless, when both family structure and cohorts fixed effects are added, the constant coefficient is not accurately estimated. While we do not enter into the methodological details that can call for a more parsimonious specification (see Schulhofer-Wohl (2018)), we consider the results reported in this section as suggestive evidence that the expenditure share on tradables decreases with age, especially after year 60, and that this relationship is robust over different cross-sections of households in different years and over different detail-levels of consumption-items categorization.

Finally, Figure 14 plots the β^k_{age} coefficients from the estimation of equation (B.1) where the dependent variable (instead of s^k_{it}) is the natural logarithm of the nominal expenditure of household i during year t on tradable and non-tradable consumption category k.[59] Nontradable expenditure grows significantly more over the life-cycle compared to tradable consumption. This pattern is consistent with the profile obtained from our overlapping generation model (see panel 1,1 of Figure 3).

[59]This is the same equation estimated by Aguiar and Hurst (2013) except that they use different consumption categories.

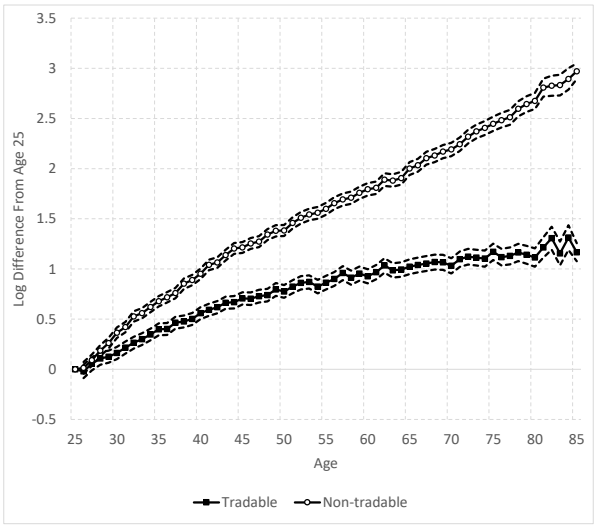

Figure 14: Life-cycle profiles of tradable and non-tradable expenditures

Note. Mean log-expenditure by age conditional on cohort, normalized year, and family status control. Each point of the continuous lines represents the coefficient on the corresponding age dummy Age_{it} from the estimation of equation (B.1), with age 25 being the omitted group, where the dependent variable (instead of s_{it}^k) is the natural logarithm of the nominal expenditure of household i during year t on consumption category $k \in \{\text{T}, \text{NT}\}$. The dashed lines around each point represent the regression coefficient plus and minus two times its standard error.

Aggregate	Variable	Tradability	Description (cf. http://www.nber.org/ces_cbo/varlist.txt)
FOOD	foodhome	T	Food Off-Premise
	foodout	NT	Food On-Premise
	foodwork	NT	Food Furnished Employees
CLOTHES & PERSONAL CARE	clothes	T	Clothing and Shoes
	toiletry	T	Toilet Articles and Preparations
	hlthbeau	NT	Barbershops, Beauty Parlors, Health Clubs
	jewelry	T	Jewelry and Watches
	tailors	NT	Clothing Services
UTILITIES	elect	NT	Electricity
	gas	NT	Gas
	water	NT	Water and Other Sanitary Services
	homefuel	NT	Fuel Oil and Coal
	telephon	NT	Telephone and Telegraph
TRANSPORTATION	carservs	NT	Repair, Greasing, Washing, Parking, Storage, Rental
	gasoline	T	Gasoline and Oil
	tolls	NT	Bridge, Tunnel, Ferry, and Road Tolls
	autoins	T	Auto Insurance
	masstran	NT	Mass Transit Systems
	othtrans	NT	Taxicab, Railway, Bus, and Other Travel Expenses
ALCOHOL & TOBACCO	tobacco	T	Tobacco Products
	alcohol	T	Alcohol Off-Premise
	niteclub	NT	Alcohol On-Premise
OTHER NONDURABLE	airfare	T	Airline Fares
	gambling	T	Pari-Mutuel Net Receipts
	charity	NT	Religious and Welfare Activities
DOMESTIC SERVICES	servants	NT	Domestic Service, Other Household Operation
ENTERTAINMENT	othrec	NT	Other Recreation Services
RENT	renthome	NT	Tenant-Occupied Nonfarm Dwellings--Rent
	homeval2	NT	Rental Equivalence of Owned Home
EXTRA	busiserv	NT	Business Services
	lifeins	NT	Expense of Handling Life Insurance
	books	T	Books and Maps
	pubs	T	Magazines, Newspapers, Other Nondurable Toys, etc.
	rentothr	NT	Other Rented Lodging
	housuppl	T	Nondurable Household Supplies and Equipment
DURABLES & hh MAINTENANCE	furnish	T	Furniture and Durable Household Equipment
	autos	T	New and Used Motor Vehicles
	parts	T	Tires, Tubes, Accessories, and Other Parts
	recsport	NT	Recreation and Sports Equipment
	ohmaint	T	Owned Housing Intermediate Goods/Services
EDUCATION	highedu	NT	Higher Education
	lowedu	NT	Nursery, Elementary, and Secondary Education
	othedu	NT	Other Education Services
HEALTH	drugs	NT	Drug Preparations
	orthopd	NT	Ophthalmic Products and Orthopedic Appliances
	doctors	NT	Physicians, Dentists, Other Medical Professionals
	hospital	NT	Hospitals
	nurshome	NT	Nursing Home
	helthins	NT	Health Insurance

Figure 15: Consumption expenditure items in the CEX data-set by Aguiar and Hurst (2013)

Note. "Health", "Education", "Durables and Household Maintenance" and "Extra" not used in Aguiar and Hurst (2013)'s main analysis.

B.2 Additional data patterns used in calibration

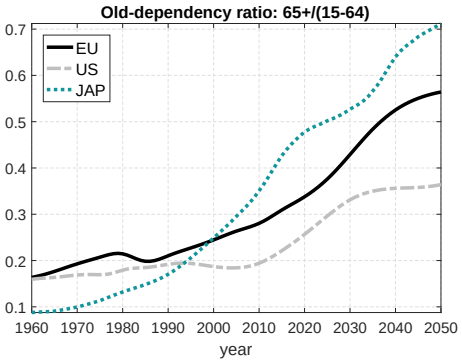

Figure 16: Old dependency ratio (Europe, United States, Japan)

Note. The old dependency ratio is defined as the number of people aged 65 or more divided by the number of people aged between 15 and 64. Source: UN (2017). Europe (EU) corresponds to 12-countries euro-zone (see footnote 31).

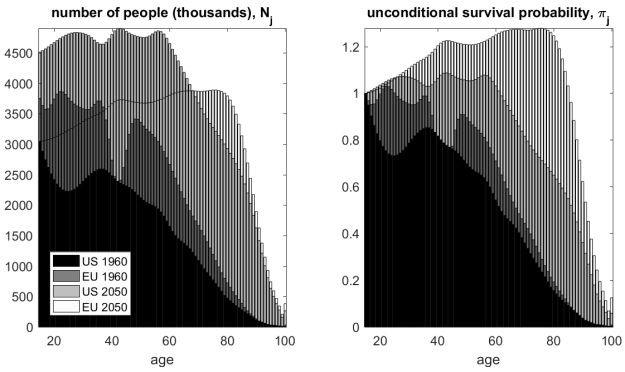

Figure 17: Demographics in the model

Note. Data on the number of people for each 1-year age-group j for each year t, $N_{t,j}$ are provided by UN (2017) (*World Population Prospects: The 2017 Revision*), medium fertility variant for year 2050, smoothed using a local regression with weighted linear least squares and a 2nd degree polynomial model using the "loess" function in Matlab with a span of 0.08. People aged more than 100 are counted together at the age-bin 100.

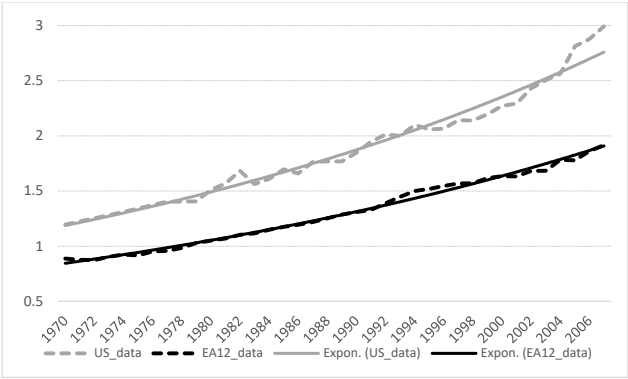

Figure 18: Data 1970-2007: relative labor productivity in the tradable sector, constantly growing over time

Note. Our own elaboration on EUKLEMS data using Cardi and Restout (2015)'s method. Labor productivity measured as "gross value added, volume indices, 1995=100" (VA_QI) over "total hours worked by persons engaged" (H_EMP). The constant yearly compound growth plotted in EU_fit is 2.226% while in US_fit is 2.3168% which are estimated by regressing the log of the relative labor productivity on time and a constant. See footnote 31 on how EU variables are constructed. Data in the period 1970-1976 for US are interpolated using the 2.3168% growth rate estimated in the time-window 1977-2007. See footnote 37 for classification of EUKLEMS industries into tradable and nontradable.

C Empirical analysis appendix

C.1 Panel cointegration analysis

C.1.1 Data

Data for all variables used in the empircal analysis in section 2, except the old dependency ratio, comes from the new dataset built by Ricci et al. (2013).[60] It is their "large sample" composed by 48 countries (21 'advanced economies' and 27 'emerging economies')[61] for the period 1980-2004. It accounts for over 90% of

[60]The dataset is not publicly available. It has been kindly shared with us by the authors.

[61](i) Advanced economies: Australia, Canada, Denmark, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom, United States; and 11 euro area countries comprising Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain. (ii) Newly industrialized or emerging markets: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Hong Kong SAR, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Russia, Singapore, Slovak Republic, Slovenia, South Africa, Tai-

Figure 19: Age dependent labor supply in efficiency units, h_j

Note. To obtain the age dependent labor supply in efficiency units we used a cubic interpolation suited for the age structure in our model on the data points provided in Domeij and Flodén (2006). These data points are the product of participation rates provided by Fullerton (1999) and productivity provided by Hansen (1993).

world trade. See their section "APPENDIX: DATA CONSTRUCTION" for details. Here a short description of the variables in Table 1:

(i) *Real effective exchange rate*. Dependent variable, based on CPI and trade weights constructed from 1999 to 2001 data (Bayoumi et al., 2006). An increase means appreciation;

(ii) *Commodity terms of trade* is the ratio of a weighted average price of the main commodity exports to a weighted average price of the main commodity imports (weights are given by the share of export/imports of a commodity by trading partners). This measure is preferred to usual terms of trade (defined as the ratio of general export to import prices indices) because is "less plagued by endogeneity problems (given that commodity prices are set on world markets and less sensitive to nominal exchange rate shocks)";

(iii) *Net foreign assets to trade* is the ratio of net foreign assets (NFAs), at the end of the previous period, to the average exports and imports (in goods and nonfactor services) of the previous period.

wan Province of China, Thailand, Turkey, and Venezuela.

(iv) *Government consumption to GDP* is the ratio of government consumption (purchases of goods and services plus government wages) to GDP;

(v) *Productivity differential* is the ratio of labor productivity (output per worker) in the tradable sector over the nontradable sector relative to trading partners;[62]

(vi) *Trade restriction index* takes the value of 0 during years of liberalization and 1 during years of restriction;

(vii) *Price controls* is the share of administered prices constructed by the EBRD 2005 (European Bank of Reconstruction and Development) out of a basket of 15 categories. Numbers are integers. It has nonzero value only for transition economies: CZE, HUN, POL, RUS, SVK, SVN.

Items (ii) to (vii) compose the main set of fundamentals that explain long-run movements in the real effective exchange rate according to Ricci et al. (2013). We add demographics as an additional fundamental, defined as:

- *Old dependency ratio relative to trading partners* ($odrw$) equals the number of people aged 65 or more over the number of people aged between 20 and 64, relative to the weighted average of the same measure for the trading partners. The measure of the number of people is provided by the United Nations *World Population Prospects: The 2015 Revision*. Consistently with the variables in Ricci et al. (2013) measured relatively to trading partners, we construct the old dependency ratio of each country i at time t relative to a geometric average of the old dependency ratios of its trading partners using time-invariant trade weights:

$$odrw_{i,t} = \frac{odr_{i,t}}{\prod_{j=1}^{J}(odr_{i,j,t})^{w_{i,j}}} \tag{C.1}$$

  where $j = 1, 2, \cdots, J$ indexes the trading partners of country i while $w_{i,j}$ are the trade weights measured on 1999-2001 data and published by the Bank

[62] Six-sector classification. The tradables sector includes: agriculture, hunting, forestry, and fishing; mining, manufacturing, and utilities; and transport, storage, and communication, whereas the nontradables sector includes: construction; wholesale and retail trade; and other services.

of International Settlements.[63] Using these weights, three countries get excluded from the "large sample" of Ricci et al. (2013) because of lack of data: Taiwan, Pakistan, Morocco.[64] We use time-invariant trade-weights for the period 1999-2001 because, as reported above, the dependent variable as well as those explanatory variables measured relatively to trading partners in Ricci et al. (2013) are constructed by using time-invariant trade-weights for the same period.

C.1.2 Panel unit root and cointegration tests

Table 8 presents Pesaran (2007)'s panel unit root test statistics accounting for cross-sectional dependence. It is a replica of Table 3 in Ricci et al. (2013) but it adds the result of the test for our main explanatory variable, i.e. the log of the old dependency ratio relative to trading partners. For all explanatory variables over the sample period there is evidence of unit root behaviour, which is a precondition for exploring a panel cointegrating relationship.

Table 9 presents the results of Pedroni (1999)'s seven panel cointegration test statistics under the null hypothesis of no cointegration in a heterogeneous panel where the regressors are the ones included in Table 8. Four out of the seven test statistics significantly reject the null hypothesis of no-cointegration thus providing evidence of panel cointegration among our variables.

C.1.3 Error correction mechanism

For each estimated cointegration relationship computed either under DOLS or CCEP in the main text, we compute the following error-correction mechanism to asses the speed of adjustment of the REER towards its long-run equilibrium:

$$\begin{aligned} \Delta \log(REER_{i,t}) &= \theta_i + \eta gap_{i,t-1} + \phi \Delta \log(REER_{i,t-1}) + \Delta X_{i,t-1}\psi + v_{i,t} \quad \text{(C.2)} \\ gap_{i,t-1} &= \log(REER_{i,t-1}) - \alpha_i - X_{i,t-1}\beta \end{aligned}$$

[63] `http://www.bis.org/statistics/eer.htm` Here, time-varying trade weights are provided too but we opted to use those applying to the period 1999-2001 to be consistent with the measures used by Ricci et al. (2013). Weights on other periods are used in section C.1.4 as a robustness check.

[64] The trade weights used in Ricci et al. (2013) were not provided. Furthermore, the United Nations World Population Prospects, 2015 Revision, does not provide demographic data for Taiwan.

Table 8: Panel Unit Root Test Statistic

Variable	Statistics
Log old dependency ratio	-1.299 *
Log (REER)	-1.926 *
Log commodity terms of trade	-1.300 *
Net foreign assets to trade	-1.262 *
Gov consumption to GDP	-1.211 *
Log productivity differential	-1.813 *

Note. The table presents Pesaran (2007)'s panel unit root test statistic in presence of cross section dependence. It is the cross-sectionally augmented IPS (Im et al., 2003) test, referred-to as CIPS. The null hypothesis is homogeneous non-stationarity (i.e. unit root). The critical values at 10%, 5%, 1% are respectively: -2.07, -2.15, -2.3. The * indicates that the null hypothesis of unit root cannot be rejected at the 10% level. The number of lags to include in each individual regression is chosen by Pesaran (2007)'s routine out of a maximum number of lags set by the econometrician. For each cross-section we set the maximum number of lags at 10. Since this test requires a balanced panel, some countries and years were dropped from the sample.

Table 9: Panel Cointegration Tests

Test Statistics	Panel	Group
v	-1.553	
rho	4.565	7.214
t	-3.705*	-4.762*
adf	-3.294*	-3.56*

Note. The table presents Pedroni (1999)'s 7 panel cointegration test statistics under the null hypothesis of no cointegration in a heterogeneous panel. The regressors included to explain log RER are 5: log old dependency ratio, log commodity terms of trade, net foreign assets to trade, government consumption to GDP, log productivity differential. All statistics are distributed as a standard normal $N(0,1)$ under the null. The relevant tail for the critical values is the negative one for all statistics except v: all statistics except v diverge to negative infinity. The * means that the null of no cointegration can be rejected at the 1% level. Following Pedroni (1999)'s terminology "Panel [cointegration statistics]" refers to the within dimension based statistics while "Group [mean cointegration statistics]" refers to the between-dimension based statistics, i.e. the former are based on estimators that pool the autoregressive coefficient across different members, while the latter average the individually estimated coefficients for each member. See Pedroni (1999) for further details. Contrary to the Panel Unit Root Test in Table 8, here the sample does not need to be balanced and gaps are allowed. Therefore we used the full "demographic sample". The tests are based on augmented Dickey–Fuller (ADF) regressions that we run with a number of lags chosen with the Akaike information criterion out of a maximum of 2.

As emphasized by Ricci et al. (2013), the long-run relationship (if any) should be interpreted as an equilibrium relationship rather than a causal one.

Table 10: ERROR CORRECTION MECHANISM (DEPENDENT VARIABLE: CHANGE IN LOG REER)

	(1) DOLS	(2) DOLS	(3) CCEP	(4) DOLS	(5) CCEP
gap$(t-1)$	-0.243***	-0.253***	-0.282***	-0.259***	-0.273***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Note. The number reported in each column is the estimate of the η coefficient associated with the variable $gap_{i,t-1}$ in error-correction mechanism (ECM) equation (C.2). Under DOLS methodology the ECM formulation is computed with Newey-West standard errors with 1 lag order of autocorrelation. Under CCEP methodology all variables in the ECM equation (C.2) are included as cross sectional averages and the estimate of η is the group mean estimate.

Table 10 shows the estimates for the coefficient η ("the speed of adjustment") in the error correction equation (C.2) under the different specifications underlying the results in Table 1. The coefficient η is: how much does REER depreciate next period if in the current period it is 1% more appreciated than its long-run value justified by fundamentals, i.e. if $gap_{i,t-1} = 1\%$. Our estimates using DOLS reported in Table 10 suggests that about 1/4 of the gap is absorbed with a change in REER within a year. When we use CCEP methodology this adjustment is somehow faster. In other terms, it takes about 4 years for REER to adjust to its long-run equilibrium according to our estimates. When the old dependency ratio is added among the fundamentals the speed of adjustment is not altered significantly (compare column (2) with (4) and column (3) with (5)). These estimates are in line with the error correction specification of Ricci et al. (2013), see their Panel B of Table 4 . Table 11 and 12 in Appendix C.1.4 report two sets of robustness checks that corroborate our estimates.

C.1.4 Robustness

We run two sets of robustness.

(i) In Table 11 we use different measures of the old dependency ratio. Given the DOLS methodology used, the estimates should be compared to Table 1.(4). In columns (1) to (4) we check how sensitive our results are to the chosen period for the trade weights. In our baseline (see Table 1) we used the trade weights

provided by the Bank of International Settlements (BIS) computed on the trade data for the period 1999-2001 to be consistent with the weights used to construct the other variables by Ricci et al. (2013). However, the BIS provides weights computed on trade data for each group of 3 years from 1995 till 2011 (relevant to us only till 2004). For each column from (1) to (3) the measure of old dependency ratio relative to trading partners has been computed using a different set of time-invariant trade weights (i.e. different $w_{i,j}$ in (C.1)) that the BIS has computed based on trade data in the following 3 years period: 1993-1995, 1996-1998, 2002-2004 respectively. We see that the coefficient on log old dependency ratio is not significantly affected by using a different set of time-invariant trade-weights as compared to the results in Table 1. In column (4) we construct the old dependency ratio with time-variant weights: in each 3-year period our measure of old dependency ratio is weighted by the weights corresponding to that period (due to lack of data, observations for the period 1980-1995 are all weighted by the 1993-1995 weights). This time the coefficient on log old dependency ratio is smaller with a p-value of 11%. But it is still positive. Moreover, the use of time-varying weights while more accurate to capture the dynamics of trade over time might be inappropriate in the context of the dataset employed here as variables have been computed using time-invariant trade weights based on the period 1999-2001 by Ricci et al. (2013). In column (5) and (6) we modify the measure of old-dependency ratio considering the number of old people (aged 65 or more) 1 and 5 years ahead instead of the contemporaneous one. The theoretical motivation for this is that the old-dependency ratio can be interpreted as a proxy for the survival probability (see two-periods model) when the number of old people in the future is taken. If people expect to live longer they might save more leading to a reduction of the equilibrium real interest rate which in turn should dampen the effect on the RER.[65] Column (5) and (6) show that this dampening effect is not there at least using this rough proxy. But it could also be that all the effects driven by saving are already captured by the variable "net foreign asset to trade" or by other variables.

(ii) As a second set of robustness checks we trim our "demographic sample"

[65] A reduction of the real interest rate tilts consumption towards younger cohorts who consume relatively more tradables than nontradables.

dropping those countries with extreme values of the log old dependency ratio relative to trading partners (see Table 12). A visual inspection reveals that Japan is the country with the greatest increase of the old-dependency ratio over the sample period, significantly outstripping all the other countries since about the median-point in time of the sample. Columns (1) and (2) reveal that the exclusion of Japan itself reduces the variability in the sample, dampening a bit the effect of aging on the real effective exchange rate: with DOLS methodology the coefficient goes from .29 of Table 1.(4) to .257, still statistically significant at the 10% level, with CCEP methodology it goes from .276 of Table 1.(5) to .135, not statistically significant. In columns (3) and (4) we drop those countries in the sample that have extreme values of the log old dependency ratio relative to trading partners according to the 5th and 95th percentile in 1992 (the median point in time in our sample). In both columns the coefficient is greater than .2 but with DOLS methodology (column (3)) is statistically insignificant. Overall, the fact that the coefficient on log old dependency ratio is always positive although not always statistically significant under this second set of robustness checks is reassuring.

Table 11: Different measures of old dependency ratio, dependent variable: log real effective exchange rate. Methodology: DOLS. Period: 1980-2004.

	(1)	(2)	(3)	(4)	(5)	(6)
lodrw9395	0.338					
	(0.004)					
lodrw9698		0.304				
		(0.010)				
lodrw0204			0.295			
			(0.013)			
lodrw_tv				0.182		
				(0.111)		
lodrw_t1					0.290	
					(0.014)	
lodrw_t5						0.290
						(0.017)
Log commodity terms of trade	0.649	0.653	0.656	0.655	0.664	0.667
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Net foreign assets to trade	0.0403	0.0405	0.0405	0.0443	0.0410	0.0392
	(0.006)	(0.006)	(0.006)	(0.003)	(0.006)	(0.008)
Gov consumption to GDP	2.610	2.674	2.712	2.755	2.698	2.746
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Log productivity differential	0.174	0.175	0.177	0.193	0.180	0.192
	(0.005)	(0.005)	(0.005)	(0.002)	(0.004)	(0.003)
Trade restriction index	0.0873	0.0946	0.1000	0.0836	0.100	0.115
	(0.143)	(0.114)	(0.094)	(0.163)	(0.092)	(0.048)
Price controls	-0.0376	-0.0378	-0.0387	-0.0371	-0.0362	-0.0385
	(0.116)	(0.111)	(0.104)	(0.120)	(0.124)	(0.123)
Number of countries	807	807	807	807	807	807

p-values in parentheses

Note. Estimates based on the "demographic sample". Methodology: DOLS, 2.1 as in Ricci et al. (2013): country fixed effects, 1 lead and 1 lag ($p = 1$), Newey-West standard errors with 1 lag order of autocorrelation. All specifications include a dummy to account for periods of capital account liberalization for Indonesia (1980-82) and Argentina (1991-2001). Each column reports estimates using a different measure of log old dependency ratio relative to trading partners ($odrw$): (1) lodrw9395 is the log $odrw$ with 1993-1995 period trade weights; (2) lodrw9698 is the log $odrw$ with 1996-1998 period trade weights; (3) lodrw0204 is the log $odrw$ with 2002-2004 period trade weights; (4) lodrw_tv is the log $odrw$ with time-varying trade weights: for observations in 1980-1995 the 1993-1995 weights are applied; for the remaining periods, 1996-2004, trade weights vary every three years; (5) & (6) for each year t lodrw_t1 (lodrw_t5) is the log of the number of people aged 65 or more in year $t + 1$ ($t + 5$) over the number of peopled aged between 20 and 64 in year t. In all cases the trade weights are provided by BIS at `http://www.bis.org/statistics/eer.htm` and refer to $w_{i,j}$ in equation (C.1).

Table 12: Robustness: trim the extremes. Dependent variable: log real effective exchange rate. Period: 1980-2004.

	No Japan		No 5-95 percentiles	
	(1)	(2)	(3)	(4)
	DOLS	CCEP	DOLS	CCEP
Log old dependency ratio	0.257	0.135	0.202	0.266
	(0.058)	(0.123)	(0.145)	(0.001)
Log commodity terms of trade	0.611	0.379	0.670	0.426
	(0.000)	(0.000)	(0.000)	(0.000)
Net foreign assets to trade	0.0388	0.0133	0.0467	0.0331
	(0.014)	(0.163)	(0.003)	(0.000)
Gov consumption to GDP	2.830	2.345	2.978	2.245
	(0.000)	(0.000)	(0.000)	(0.000)
Log productivity differential	0.177	0.223	0.231	0.259
	(0.005)	(0.000)	(0.000)	(0.000)
Trade restriction index	0.103	0.128	0.0796	0.181
	(0.088)	(0.000)	(0.329)	(0.000)
Price controls	-0.0372	-0.0336	-0.0403	-0.0278
	(0.115)	(0.054)	(0.088)	(0.119)
Number of countries	786	918	719	842

p-values in parentheses

Note. DOLS and CCEP methodologies employed follow the same specifications used in Table 1, cf. note there. Estimates based on the "demographic sample" with the following modifications: in column (1) and (2) Japan has been excluded from the sample; in column (3) and (4) countries are excluded from the sample according to the following criterion: exclude those countries whose log old dependency ratio in 1992 is either smaller than its 5th percentile or grater than its 95th percentile.

C.1.5 Percentage contribution to absolute deviation

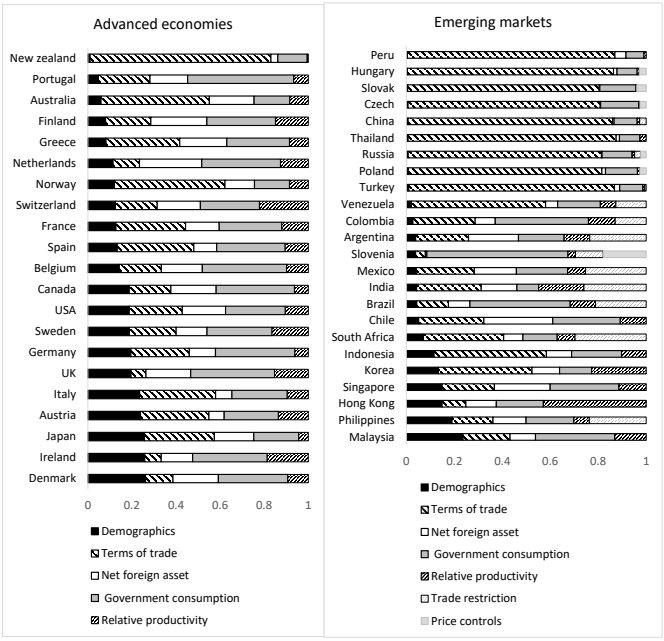

Figure 20: Predicted REER: mean % contribution to absolute deviation

Note. Country-specific values underlying Figure 8. Groups: (a) *Advanced countries*: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States; (b)*Emerging markets*: China, Hong Kong SAR, India, Indonesia, Korea, Malaysia, Philippines, Singapore, Thailand, Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela, Czech Republic, Hungary, Poland, Russia, Slovak Republic, Slovenia, South Africa, Turkey.